

Optimal Fiscal Limits with Overrides*

Abstract

This paper studies the optimal design of fiscal limits in the context of a simple political economy model. A politician chooses the level of taxation for a representative citizen but is biased in favor of higher taxes. A constitutional designer sets a tax limit before the citizen's preferred level of taxation is fully known. The politician is allowed to override the limit with the citizen's approval. The paper solves for the optimal limit and explains how it is impacted by the possibility of overrides. The paper also shows that the citizen's welfare can be enhanced if the designer imposes a limit on the politician's override proposals.

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Stephen Coate
Department of Economics
Cornell University
Ithaca NY 14853
sc163@cornell.edu

Ross Milton
Department of Economics
Kansas State University
Manhattan, KS 66506
rmilton@ksu.edu

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1 Introduction

Despite being democratically elected, most state and local governments in the US cannot simply set fiscal policies as they see fit. Rather, tax and spending limits are a common feature of the state and local government fiscal landscape. At the state level, Rose (2010) and Waisanen (2010) report that 30 state governments operate under a tax or spending limitation. In some states these limits are constitutional and in others they are statutory. They have been implemented both by state legislative bodies and directly by citizens through the initiative process. At the local level, Mullins (2010) reports that all but 3 states impose some form of constitutional or statutory statewide limitation on the fiscal behavior of their local governments. Moreover, self-imposed limits are quite prevalent at the local level as documented by Brooks, Halberstam, and Phillips (2016).

Limits come in many forms and apply to a variety of different fiscal variables. There are limits on tax rates, tax revenues, and also on government spending. At the local level, limits on property taxes, the predominant source of local revenue, are very common. However, the limits are rarely absolute. Most contain an override provision which specify when limits may be violated – typically with either direct approval of a majority of citizens in a referendum or a super-majority vote of the governing legislative body. In the case of local taxes, of the 42 US states that have a limit, 35 allow local governments to override some or all of their limitations with the approval of a majority of their voters.¹

In light of their practical significance, it is interesting to consider what principles might guide the designers of fiscal constitutions in the setting of fiscal limits and the use of overrides. This requires studying the optimal design of limits. To analyze the problem, it is first necessary to frame it. This demands taking a stand on why limits are needed and specifying the constraints constitutional designers face in setting them. Regarding the former, it seems natural to assume that elected politicians tend to have a bias towards larger government relative to their constituents.²

Regarding the latter, it is clear that, at the time of setting a limit, constitutional designers

¹ This information is obtained from Lincoln Institute for Land Policy (2016) and Mullins (2003), (2010). Of the 7 states with limits that do not allow voter overrides, 2 (Indiana, South Carolina) allow overrides of the limit with supermajority votes of the governing body or with an appeal to the state government and the remaining 5 (Alaska, Arkansas, Delaware, Utah, and Wyoming) provide no means for local governments to exceed the limit.

² This assumption seems consistent with the history of local tax limitations. Most states adopted their current property tax limits in the late 1970's and early 1980's during an anti-tax movement following the passage of Proposition 13 in California (Mullins 2010). The literature examining the causes of this 'revolt' commonly credits a perception among voters that taxes were too high, despite being set by democratically elected governments (see, for example, Citrin 1979 and Ladd and Wilson 1982). Why elected politicians might be biased towards larger

face uncertainty in what citizens' preferred fiscal policies are going to be. This uncertainty is what motivates designers limiting politicians' fiscal authority rather than simply requiring they implement specific levels of taxation and spending.

With the problem framed in this way, it is natural to look to the literature on the *delegation problem* for guidance on optimal fiscal limits.³ This literature considers the interaction between a principal and an agent. The agent has to choose a policy that impacts both the principal and agent's payoffs. The optimal policy for both principal and agent depends on the realization of a state of nature, which, prior to the policy choice, is only observed by the agent. The agent's payoff differs from that of the principal, being biased towards a higher or lower level of the policy. The key assumption of the literature is that policy-contingent transfers between the principal and agent are not possible. Rather, the nature of the interaction is assumed to be that the principal chooses a set of permissible policies for the agent and, given his information on the state of nature, the agent chooses his preferred policy from this set. Thus, the choice is delegated to the agent, but the principal places limits on the agent's discretion. The analytical question of interest is what is the optimal set of permissible policies from the principal's perspective? The literature finds reasonable conditions under which the optimal set is an interval and explores how this interval depends on features of the underlying environment such as the extent of the agent's bias and the degree of uncertainty in the state of nature.

To apply these results to the optimal design of fiscal limits, the principal is interpreted to be the constitutional designer and the agent to be a politician.⁴ The policy is the level of spending or taxation which impacts the welfare of a representative citizen. The politician is assumed to be biased in favor of larger government relative to the representative citizen whose interests the designer pursues. The state of nature represents things that impact the citizen's and politician's preferred level of spending or taxation that are uncertain at the time at which the limit is set. Under the conditions that imply the optimal set of permissible policies is an interval, the upper bound of this interval will be the optimal fiscal limit. The determinants of the optimal limit then follow from the determinants of the optimal interval.

government is discussed further in the On-line Appendix.

³ The literature on the delegation problem is a sub-branch of the literature on contract theory. It includes Alonso and Matouschek (2008), Amador and Bagwell (2013), Amador, Werning, and Angeletos (2006), Holmstrom (1977), (1984), and Melumad and Shibano (1991). The theory has many interesting applications, including several in the field of political economy. One such application is to the delegation of policy-making from elected politicians to bureaucrats (see, for example, Epstein and O'Halloran 1994 and Huber and Shipan 2006). Another is to the delegation of policy-making from legislatures to standing committees (see, for example, Gilligan and Krehbiel 1987, 1989, and Krishna and Morgan 2001).

⁴ The application to "optimal fiscal constitutions" is noted explicitly by Amador, Werning, and Angeletos (2006).

There is, however, a discrepancy between this way of formalizing the problem and the way in which fiscal limits are structured in reality: namely, the presence of overrides. The formalization does not allow the politician to deviate from the permissible set of policies with the representative citizen's approval.⁵ It is therefore natural to wonder how this impacts the applicability of the results of the delegation literature to the optimal design of fiscal limits. Of course, one possibility is that, while overrides are in principle available, in practice they are hardly ever used, and therefore it is reasonable to simply assume them away. Below we present evidence from 3 US states on local government property tax limits that this is far from the case. For example, in 2014, 18% of Wisconsin school districts, 4% of Massachusetts municipalities, and 11% of Ohio local governments overrode their tax limits. Given this, it seems important to incorporate overrides into the analysis and explore how their presence impacts the optimal design of fiscal limits. This is precisely the purpose of this paper.

To introduce overrides, the paper assumes that after the state of nature has been revealed, the politician can propose a taxation or spending level that violates the limit. If the representative citizen votes for the politician's proposal, it is implemented. If he votes against, the proposal is not implemented and the politician is required to select an alternative policy that respects the limit. At the time of voting, the citizen is assumed to know the state of nature, meaning that he knows his optimal policy.⁶ However, since the politician has agenda-setting power, this does not imply that the citizen gets to enjoy this policy.

The paper solves for the optimal limit under this assumption concerning overrides. It explores how the optimal limit depends upon the extent of politician bias and explains how the optimal limit with overrides compares to that without. It also considers the optimality of this particular institutional arrangement, seeking to identify reforms that can improve the citizen's welfare.

There are three main results. First, with overrides, the optimal limit is at least as stringent

⁵ In the delegation problem setting, Mylovannov (2008) shows that the principal can implement an optimal outcome through *veto-based delegation* with an appropriately chosen default policy. In this implementation, the agent proposes a policy and then the principal approves it or not. If he does not approve it, the default policy is implemented. However, this differs from an override which requires that the politician obtains the citizen's approval only if he exceeds the limit. Moreover, in Mylovannov's scheme the principal is not fully informed when he is voting on the agent's proposal. Rather he makes inferences about what must be true from the agent's proposal.

⁶ This may appear similar to the set-up of Epstein and O'Halloran (1994). In their well-known model, a politician must decide how much discretion to provide to a bureaucrat. The bureaucrat makes a policy proposal within an interval of permissible proposals set by the politician. After the bureaucrat has made his proposal, the politician may veto it. If he does so, some default policy is implemented. As in our model, at the time of the veto decision, the politician is fully informed. However, this model differs from ours in that i) the bureaucrat's proposal is always subject to the politician's veto, and ii) the bureaucrat cannot choose from outside the interval of permissible policies. In our model, the voter (who corresponds to the politician in their model) only votes if the politician (who corresponds to the bureaucrat) proposes something outside the permissible set.

as without. The two limits coincide in a broad set of circumstances but differ at low levels of politician bias for some distributions of the state of nature. They coincide when, under the optimal limit with overrides, the politician always exploits his agenda-setting power when making override proposals to leave the citizen with the same utility as he would get with policy at the limit. Under these circumstances, while equilibrium will involve overrides, the citizen's utility will be the same as if there were no overrides. As a consequence, the optimal limit is the same with or without overrides. This scenario necessarily arises when the politician's bias is sufficiently large. For smaller levels of bias, however, the citizen may strictly prefer the politician's proposal to the limit. When this happens, the optimal limit with overrides is more stringent. Intuitively, the cost of a tighter limit is reduced because of the flexibility permitted by overrides.

Second, with overrides, the typical monotonic relationship in delegation models between the agent's bias and the tightness of the limit does not necessarily arise. The reason is that there is now a countervailing force. As bias declines, the value of overrides rises, since the politician and citizen are more in agreement when the politician makes a proposal. To increase the chance of a beneficial override ex post, it can actually be optimal to have a tighter limit for a less biased politician.

Third, the institutional arrangement consisting of a limit and an override provision can always be strictly dominated by an arrangement that also specifies an override limit. Limiting the proposal the politician can make at the override stage prevents him from fully exploiting his agenda-setting power. The availability of an override limit changes the calculus underlying the regular limit. Since a limit increases the benefit of overrides, the regular limit is often tighter, making overrides more likely.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 presents evidence of the empirical importance of the override provision from the US states. Section 4 outlines the model and characterizes the optimal limit. Section 5 compares the optimal limit with and without overrides. Section 6 demonstrates the benefits of an override limit and characterizes the optimal system of limits. Section 7 presents some examples and Section 8 concludes.

2 Related literature

The literature on fiscal limits can be divided into four branches.⁷ The first documents the types of limits faced by state and local governments in the US and describes when and how they were introduced (see, for example, Mullins 2010 and Waisanen 2010). This is a difficult task because there is considerable variation across states and localities and significant change over time. The second branch is devoted to understanding how limits impact the fiscal variables they seek to regulate and other related public policies.⁸ This is challenging because of the problem of identifying the effect of limits. The third branch studies what citizens think about existing limits and why they were introduced (see, for example, Citrin 1979, Courant, Gramlich and Rubinfeld 1985, Cutler, Elmendorf, and Zeckhauser 1999, and Ladd and Wilson 1982). The fourth branch addresses the normative question of whether limits enhance citizens' welfare and, if so, what should be limited and how should limits be designed.

This paper contributes to this fourth, normative branch of the literature. Other papers in this branch are Brennan and Buchanan (1979), Besley and Smart (2007), and Brooks, Halberstam, and Phillips (2016). Brennan and Buchanan (1979) consider tax limits in the context of a Leviathan government that would like to maximize revenues raised. They argue that assessing the appropriate tax limit will be too complicated for average citizens. Our analysis seeks to provide guidance on exactly this type of question.

Besley and Smart (2007) study the operation of a tax revenue limit in the context of a two period political agency model in which politicians can be good or bad. The important point that Besley and Smart make is that a revenue limit impacts how much voters learn about the incumbent. In particular, a revenue limit might prevent voters from distinguishing between good and bad politicians in the first period, leading to worse outcomes in the second period. Our analysis abstracts from this interesting point by assuming that the politician's bias is independent of the fiscal limit.

As a prelude to their empirical work, Brooks, Halberstam, and Phillips (2016) provide a theoretical analysis of optimal limits that is in the spirit of this paper. Their framework for understanding limits builds on a model of local government elections presented in Coate and

⁷ Selective reviews of the literature are provided by Krol (2007), Mullins and Wallin (2004), and Rose (2010).

⁸ Papers in this branch include Bradbury, Mayer, and Case (2001), Brooks, Halberstam, and Phillips (2013), Dye and McGuire (1997), Figlio and Rueben (2001), Knight (2000), Poterba (1994), and Poterba and Rueben (1995, 2001). After surveying the literature, Rose (2010) concludes "tax and expenditure limits appear to be modestly effective in slowing the growth of government, particularly at the municipal level, although the evidence is somewhat mixed".

Knight (2011). Citizens differ in their preferences for public spending. When low preference politicians hold office, they implement a limit to constrain future high spenders. The benefits of public spending are uncertain which makes the choice of limit non-trivial. The optimal limit is more permissive the higher the probability elected politicians are low preference types. While it does not explicitly incorporate elections, our model is consistent with that of Brooks, Halberstam, and Phillips. However, our analysis of optimal limits differs from theirs by incorporating the reality that limits can be overridden, which changes the calculus of the optimal limit.

More generally, the paper contributes to a broader normative literature on fiscal constitutions. A fiscal constitution is a set of rules and procedures that govern the determination of fiscal policies (see, for example, Brennan and Buchanan 1980). It is distinct from a political constitution which sets up the architecture of government and the rules by which policy-makers are selected. The fiscal constitution literature seeks to understand the effectiveness of various rules and procedures in generating good fiscal policies for citizens. In addition to tax and spending limits, it studies balanced budget rules, budgetary procedures, debt limits, and rainy day funds. Rose (2010) provides a review of this literature.

A branch of the literature on fiscal constitutions that is currently very active is that concerning fiscal rules to deal with rising government debt (see Yared 2018 for an overview). Azzimonti, Battaglini, and Coate (2016) study the costs and benefits of imposing a balanced budget rule in a dynamic political economy model of fiscal policy. In a series of papers, Halac and Yared explore the problem of designing deficit rules using a mechanism design approach (Halac and Yared 2014, 2016, 2017, and 2018). Their underlying model, which builds on Amador, Werning, and Angeletos (2006), assumes that politicians have time inconsistent preferences and that the benefits of current spending are ex ante uncertain. The mechanism design approach taken by Halac and Yared differs from that followed in this paper, which treats the institutional arrangement of a limit and override provision as given and analyzes what determines the stringency of the limit. Nonetheless, in studying how this arrangement can be dominated by a reform that imposes a limit on override proposals, this paper is influenced by their work. Halac and Yared (2016) is particularly related to this paper in that it focuses on the optimal use of escape clauses in deficit rules. Halac and Yared model an escape clause where, if the politician requests it, the constitutional designer will verify the state of nature and implement the optimal policy. Since verification is costly, it is employed only in special circumstances. When costs are relatively small, the optimal arrangement involves politicians requesting verification when the benefit of spending is very high and otherwise

respecting a deficit limit. Requesting verification and holding an override election both represent escape clauses, but with a substantial difference. In the model of this paper, a successful override does not implement the optimal policy but rather the politician’s proposed policy. Even though the politician’s proposal must be approved by the citizens, it will not equal the optimal policy because the politician has agenda-setting power.

Finally, the model studied here is related to the well-known agenda-setter model of Romer-Rosenthal (Romer and Rosenthal 1978 and 1979). The agenda-setter model considers the interaction between a politician and a representative voter. The voter’s utility depends on the level of public spending and the politician is responsible for choosing the level of this spending. The politician is a budget maximizer and hence maximally biased in favor of spending. The politician’s proposed spending level must be approved by the voter and, if it is not, an exogenous reversion level is implemented. In equilibrium, the politician proposes a spending level which leaves the voter indifferent between the proposal and the reversion level. The proposed spending level exceeds the reversion level whenever the latter falls below the voter’s preferred spending level. In this paper, the choice of the limit can be thought of as endogenizing the reversion level.

3 The override provision in practice

This section illustrates the significance of overrides in practice. It focuses on local property tax limits, the most prevalent type of fiscal limit in the US. As mentioned previously, 35 states have property tax limits that allow local governments to override their limitation with the approval of a majority of their voters (Lincoln Institute for Land Policy 2016). However, the existence of override provisions does not necessarily mean that they are used. To provide a feel for how overrides are used in practice, we have assembled evidence from 3 states: Massachusetts, Ohio, and Wisconsin. We selected these states because they have data available detailing the tax limits in particular jurisdictions, taxes levied, and overrides of the limits.⁹ Wisconsin has separate tax limits for school districts and counties and municipalities, giving us 4 different cases. As we will see, in 3 of the 4, overrides are commonplace.

The details of these limits are described in the On-line Appendix, but they all share the feature that, from the perspective of local officials setting taxes for the coming year, there is a maximum revenue level they can select through their action alone. They are free to set the tax level at any

⁹ It is certainly possible that the availability of data is related to the widespread use of overrides in these states, making them unrepresentative. We doubt this is the case, but have no way to rule it out.

amount below this. In order to exceed this level, they must propose and announce an alternative level and allow the electorate to vote on an override measure. If the override vote fails they must abide by the limit. If it prevails, they may increase taxes up to that level. This is all in line with the model presented in the next section.

Interestingly, in Massachusetts, elected officials are also limited in what they can propose for an override. An absolute maximum tax rate cannot be exceeded even with the voters' approval. Since the year 2000, only 4 out of the 351 municipalities have ever taxed at this limit. In contrast in Wisconsin and Ohio, overrides are limited only by what the voters will approve. The potential value of such an override limit is discussed in Section 6.

In 3 of the 4 cases, many local governments override their limits. Between 2000 and 2016, 49% of Massachusetts municipalities, 78% of Ohio taxing jurisdictions, and 55% of Wisconsin school districts overrode their limit at least once. In contrast, between 2009 and 2016, only 18% of Wisconsin counties and municipalities approved an override.¹⁰

In the model developed in this paper, optimal limits have implications for two statistics that can be determined from the data. These are, the frequency that overrides occur and the frequency with which the government sets the policy at the limit without an override. The model suggests interpreting the former as how often both the citizens and the politicians prefer a tax level higher than the limit, while the latter describes how often politicians prefer a tax level higher than the limit but the citizens do not. Between 2000 and 2016, overrides occurred 8.7%, 10.1%, and 6.5% of the time on average in Massachusetts, Ohio, and Wisconsin school districts respectively. In Wisconsin counties and municipalities between 2009 and 2016, overrides occurred 3.45% of the time. In Massachusetts, on average, municipalities taxed at their limit 44% of the time between 2000 and 2016. In Ohio, local governments nearly always tax at the maximum level they are allowed when they do not pass an override.¹¹ On average, Wisconsin school districts set taxes at their limit 70% of the time over the same span. Between 2016 and 2017 Wisconsin counties and municipalities taxed at their limit 72% of the time. We will return to these statistics once we have explained the implications of the model.

The structure of the four limits substantially diverge in how each government's limit evolves from year to year and how that depends on their policy choices. The model in this paper is static

¹⁰ The limited years of data for Wisconsin counties and municipalities is due to the limited data obtained from the state Department of Revenue. From 2009-2016, 28%, 55%, and 34% of Massachusetts municipalities, Ohio taxing jurisdictions, and Wisconsin school districts respectively, approved at least one override.

¹¹ This may be due to the stringency of the limit or the way the policy is structured which we discuss in the On-line Appendix.

and thus ignores such dynamic considerations. Nonetheless, they are interesting, so we present a brief description of them here (further details are in the on-line appendix). The cases differ in three main ways. First, do overrides increase the limit for all subsequent years or only for a limited time? In Massachusetts, all overrides are permanent in the sense that the proposed new tax level becomes the new limit. By contrast, in Ohio and Wisconsin, local officials proposing an override can choose between a permanent and temporary increase. When a temporary override expires, the limit remains at its prior level. Second, is the limit the following year independent of the current year's policy choice in the absence of an override? In Massachusetts, Ohio, and for Wisconsin school districts it is. However, Wisconsin municipalities that do not tax up to their limit will have a lower limit the following year than if they had. Third, how do the limits increase from year to year in the absence of a new or expiring override? In all cases, the limits increase by an allowed amount for new construction (or new students). In Ohio, that is the only increase, while in Massachusetts and Wisconsin there are increases intended to adjust for inflation although they are not directly related to any measure of costs.

4 Optimal fiscal limits

4.1 The model

A politician is in charge of selecting a level of taxation for a community. A representative citizen has to pay the tax but benefits from the spending it finances. The citizen desires a certain level of taxation, but this preferred tax is ex ante uncertain. The politician prefers a higher tax than the citizen. A constitutional designer who represents the citizen is aware of the politician's bias and, before the citizen's preferred tax is revealed, imposes a tax limit on the politician. The limit comes with an override provision that allows the politician to violate it with the citizen's approval.

The tax is denoted t . The citizen's preferred tax is τ . The citizen has distance policy preferences $-|t - \tau|$ so that his utility declines linearly and symmetrically as the tax diverges in either direction from his ideal.¹² The citizen's preferred tax τ is the realization of a random variable with range $[\underline{\tau}, \bar{\tau}]$ and cumulative distribution function $H(\tau)$. The associated density function, $h(\tau)$, is assumed to be symmetric around the mean $\tau_m = (\underline{\tau} + \bar{\tau})/2$.¹³ In addition, the density is continuous and

¹² While not without their drawbacks (see, for example, Milyo 2000), distance preferences are very widely used in the political economy literature. They are both analytically tractable and simple to understand. The particular form used here assumes a linear loss of utility for the voter as the tax diverges from his ideal. This is distinct from a quadratic loss which is also commonly assumed. The On-line Appendix shows that similar results arise under this specification.

¹³ This means that for any τ below the mean τ_m , $h(\tau)$ is equal to $h(2\tau_m - \tau)$.

non-decreasing on $[\underline{\tau}, \tau_m]$. These assumptions imply that the cumulative distribution function is convex on the interval $[\underline{\tau}, \tau_m]$ and concave on the interval $[\tau_m, \bar{\tau}]$. The politician has preferred tax $(1 + b)\tau$ and preferences $-|t - (1 + b)\tau|$ so that the parameter b measures the magnitude of the politician's bias.¹⁴

The tax limit is denoted ℓ . It prevents the politician from implementing a tax in excess of ℓ without the citizen's approval.¹⁵ Without loss of generality, ℓ is assumed to belong to the interval $[\underline{\tau}, \bar{\tau}]$.

The timing of the interaction between the constitutional designer, politician, and citizen is as follows. First, knowing H and b , the designer selects a limit ℓ from the interval $[\underline{\tau}, \bar{\tau}]$. Second, nature selects the citizen's preferred tax τ which is observed by both the politician and citizen. Third, the politician proposes a tax t . If the proposal does not exceed the limit, it is implemented. Fourth, if the proposed tax violates the limit, an election is held and the citizen votes in favor or against the proposal. If he votes in favor, the proposal is implemented. Fifth, if the citizen votes against the proposal, the politician chooses another tax t' which respects the limit and this is implemented.¹⁶

4.2 The limit design problem

Now consider the designer's problem of choosing a tax limit to maximize the citizen's expected welfare. We first derive the policy implications of any given limit ℓ . If the politician had to satisfy the limit, he would choose a tax equal to the limit if this is smaller than his preferred tax $(1 + b)\tau$. Otherwise, he would choose his preferred tax. Thus his policy choice would be $\min\{\ell, (1 + b)\tau\}$. The citizen will therefore realize that if he votes down any alternative policy proposed by the politician, the tax implemented would be $\min\{\ell, (1 + b)\tau\}$. If the citizen's preferred tax τ is less than ℓ he will prefer this policy to any higher level and there is no point in the politician proposing to violate the limit. In this case, therefore, the implemented tax will be $\min\{\ell, (1 + b)\tau\}$. If τ

¹⁴ This assumption means that the politician's bias is larger in absolute terms the larger is the voter's preferred level. An alternative would be to assume that the politician's preferred level is $\tau + b$ and the bias is constant. The On-line Appendix shows that similar results arise under this specification.

¹⁵ We ignore issues of enforcement, effectively assuming that the constitutional designer has sufficient penalties available to deter the politician from violating the limit without the citizen's approval. Limits are enforced in our examples by state courts. An interesting question to ask is how limited enforcement penalties impact the limit design problem. This is addressed in the context of deficit limits using a mechanism design approach by Halac and Yared (2017).

¹⁶ The model raises at least four obvious questions which we address in the On-line Appendix. First, why cannot citizens elect a candidate who shares their tax preferences? Second, how does the constitutional designer know the politician's exact bias? Third, why is there uncertainty about the citizen's preferred tax? Finally, why does the citizen not get his preferred tax once uncertainty is resolved?

exceeds ℓ , the citizen will support taxation in excess of the limit. The optimal policy proposal for the politician solves the standard agenda-setter's problem

$$\begin{aligned} \max_{\ell} & -|t - (1+b)\tau| \\ \text{s.t.} & -|t - \tau| \geq -|\ell - \tau|. \end{aligned} \quad (1)$$

The constraint guarantees that the citizen supports the proposal since the politician will choose tax ℓ if his proposal is rejected. From the constraint, the maximum tax the citizen will support is $2\tau - \ell$. The politician will propose this if it is smaller than his preferred tax $(1+b)\tau$. Otherwise, he will choose his preferred tax. In this case, therefore, the implemented tax will be $\min\{2\tau - \ell, (1+b)\tau\}$.

Pulling all this together, it follows that with limit ℓ , the citizen's expected welfare will be given by

$$V(\ell) = - \int_{\underline{\tau}}^{\ell} [\min\{\ell, (1+b)\tau\} - \tau] h(\tau) d\tau - \int_{\ell}^{\bar{\tau}} [\min\{2\tau - \ell, (1+b)\tau\} - \tau] h(\tau) d\tau. \quad (2)$$

The designer's problem is to choose a limit ℓ from the interval $[\underline{\tau}, \bar{\tau}]$ to maximize this function.

Two conditions shape the nature of the solution. The first is whether the limit is greater or less than $(1+b)\underline{\tau}$. This determines the policy the politician chooses if τ is less than ℓ . In particular, if ℓ is less than $(1+b)\underline{\tau}$, then $\min\{\ell, (1+b)\tau\} = \ell$ if $\tau \in [\underline{\tau}, \ell]$, so that the limit is always binding. By contrast, if ℓ is greater than $(1+b)\underline{\tau}$, then

$$\min\{\ell, (1+b)\tau\} = \begin{cases} (1+b)\tau & \text{if } \tau \in [\underline{\tau}, \frac{\ell}{1+b}] \\ \ell & \text{if } \tau \in [\frac{\ell}{1+b}, \ell] \end{cases}, \quad (3)$$

so that the limit binds only if τ exceeds $\ell/(1+b)$. The second condition is whether the limit is greater or less than $(1-b)\bar{\tau}$. This determines the policy chosen if τ exceeds ℓ . If ℓ exceeds $(1-b)\bar{\tau}$, then $\min\{2\tau - \ell, (1+b)\tau\} = 2\tau - \ell$ if $\tau \in [\ell, \bar{\tau}]$, so that the citizen is left indifferent between the policy chosen and the limit ℓ . If ℓ is less than $(1-b)\bar{\tau}$, then

$$\min\{2\tau - \ell, (1+b)\tau\} = \begin{cases} 2\tau - \ell & \text{if } \tau \in [\ell, \frac{\ell}{1-b}] \\ (1+b)\tau & \text{if } \tau \in [\frac{\ell}{1-b}, \bar{\tau}] \end{cases}, \quad (4)$$

so that the citizen gets a policy that he strictly prefers to the limit when τ exceeds $\ell/(1-b)$.

4.3 The optimal limit with large politician bias

When the politician's bias is large, the limit design problem is straightforward to solve. In particular, suppose that it were the case that b exceeded $(\bar{\tau} - \underline{\tau})/\underline{\tau}$. Then ℓ would be less than $(1+b)\underline{\tau}$

for any limit in the range $[\underline{\tau}, \bar{\tau}]$. Moreover, ℓ would be greater than $(1 - b)\bar{\tau}$ for any limit in the range $[\underline{\tau}, \bar{\tau}]$. Accordingly, from (2), the objective function would be

$$V(\ell) = \int_{\underline{\tau}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau. \quad (5)$$

Differentiating this expression, we obtain

$$V'(\ell) = -H(\ell) + 1 - H(\ell). \quad (6)$$

Setting this derivative to zero, reveals that the optimal limit is equal to τ_m - the *median* preferred tax.

The intuition is straightforward. When the politician's bias exceeds $(\bar{\tau} - \underline{\tau}) / \underline{\tau}$, whatever the limit, he will always choose a tax equal to the limit when the citizen's preferred tax is less than the limit ($\tau < \ell$). Moreover, when the citizen's preferred tax exceeds the limit ($\tau > \ell$), the politician will choose a tax that provides the citizen with exactly the same payoff as he would get from the limit. As a result, the citizen's payoff is exactly that which would arise if the policy were just set equal to the limit. Now take any limit ℓ and consider marginally increasing it. This will cost the citizen whenever $\tau < \ell$ and will benefit the citizen whenever $\tau > \ell$. Moreover, because the citizen has linear distance preferences, the cost and benefit will both be equal to the change in the limit, when they occur. Accordingly, the marginal increase will benefit (harm) the citizen if the likelihood that $\tau < \ell$ is less than (exceeds) the likelihood that $\tau > \ell$. The optimal limit is therefore the limit at which the likelihood that $\tau < \ell$, which is $H(\ell)$, equals the likelihood that $\tau > \ell$, which is $1 - H(\ell)$.

As the following proposition shows, we can weaken the requirement on bias and preserve the conclusion that the optimal limit is the median preferred tax.

Proposition 1 *If the politician's bias b exceeds $(\bar{\tau} - \underline{\tau}) / 2\underline{\tau}$, the optimal limit is τ_m .*

To understand this result, note that with a level of bias between $(\bar{\tau} - \underline{\tau}) / 2\underline{\tau}$ and $(\bar{\tau} - \underline{\tau}) / \underline{\tau}$, it remains the case that the limit τ_m will cause the politician to always set the policy at the limit or propose an override that makes the citizen no better off than at the limit. However, for sufficiently large or small limits this will not be the case. The proof shows that for any such limit, moving the limit marginally towards τ_m will benefit the citizen.¹⁷

¹⁷ The proofs of Propositions 1-5 can be found in the Appendix. The proofs relating to the results in Section 6 are in the On-line Appendix.

4.4 The optimal limit with small politician bias

We now turn to the more challenging case in which b is smaller than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$. We first establish a useful preliminary result.

Lemma 1 *The optimal limit is always at least as big as τ_m .*

This involves showing that for any limit ℓ smaller than τ_m , the benefits of marginally increasing it always exceeds the costs. We already know this to be the case if ℓ is less than $(1+b)\underline{\tau}$ and greater than $(1-b)\bar{\tau}$. The proof extends the argument to the cases in which one or more of these inequalities do not hold.

We next characterize the solution when the politician's bias is between $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$ and $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$. In this range, the limit τ_m will be greater than $(1-b)\bar{\tau}$ but also greater than $(1+b)\underline{\tau}$. This means that the politician would choose his preferred tax $(1+b)\tau$ rather than τ_m for τ less than $\tau_m/(1+b)$. This will also be true for any limit larger than τ_m . Given the optimal limit is at least as large as τ_m , the benefit of marginally increasing the limit is the likelihood that $\tau > \ell$, while the cost is the likelihood that $\tau \in [\ell/(1+b), \ell)$. The optimal limit is therefore where $H(\ell) - H(\ell/(1+b))$ is equal to $1 - H(\ell)$.

Proposition 2 *If the politician's bias b lies between $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$ and $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$, the optimal limit solves the equation*

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = 1 - H(\ell). \quad (7)$$

The proof of this Proposition shows that there must exist a solution to equation (7) on the interval $(\tau_m, \bar{\tau}]$. While there is no guarantee that this solution is unique, it is difficult to come up with examples in which there are multiple solutions.¹⁸ In the case covered by Proposition 2, the optimal limit becomes more stringent as the politician's bias increases. An increase in b shifts up the curve $H(\ell) - H(\ell/(1+b))$. If there is a single intersection point, it must shift to the left, implying a lower optimal limit.¹⁹

In the case covered by Proposition 2, the optimal limit has an interesting empirical implication. The probability an override occurs must equal the probability the policy is equal to the limit. The former probability is $1 - H(\ell)$. The latter is $H(\ell) - H(\ell/(1+b))$, reflecting that the politician

¹⁸ A sufficient condition for uniqueness is that for all $\tau \geq \tau_m$ $(1+b)2h(\tau) > h(\tau/(1+b))$, which requires that the density be relatively flat.

¹⁹ If there are multiple intersection points, both the smallest and largest shift to the left. The only way that an increase in b could increase the optimal limit, therefore, is if it caused a shift from the smallest intersection point to the largest. But it is easy to see that an increase in b makes a move from the smallest intersection point to the largest less attractive.

chooses his preferred tax rather than the limit for τ less than $\ell/(1+b)$. Condition (7) implies the equality of these two probabilities. In fact, this implication also holds in the case covered by Proposition 1, where these probabilities are each $\frac{1}{2}$. Given that these statistics are in principle observable, this suggests a simple test for the optimality of a limit, at least for the range of bias covered by these cases.

Finally, we tackle the case in which the politician's bias is smaller than $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$. In this range of bias levels, the limit τ_m will be greater than $(1+b)\underline{\tau}$ and smaller than $(1-b)\bar{\tau}$. Higher limits will obviously also be greater than $(1+b)\underline{\tau}$ but could be greater or less than $(1-b)\bar{\tau}$. If the optimal limit is greater than $(1-b)\bar{\tau}$, the logic underlying Proposition 2 continues to apply and it must satisfy equation (7). If it is smaller, the benefit of marginally increasing the limit is the likelihood that $\tau \in (\ell, \ell/(1-b)]$ while the cost is the likelihood that $\tau \in [\ell/(1+b), \ell)$. The optimal limit is therefore where $H(\ell) - H(\ell/(1+b))$ is equal to $H(\ell/(1-b)) - H(\ell)$.

Proposition 3 *If the politician's bias b is smaller than $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$, the optimal limit is **either** bigger than $(1-b)\bar{\tau}$ and solves equation (7) **or** is smaller than $(1-b)\bar{\tau}$ and solves the equation*

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = H\left(\frac{\ell}{1-b}\right) - H(\ell). \quad (8)$$

As noted above, there must exist a solution to equation (7) on the interval $(\tau_m, \bar{\tau}]$. The proof of Proposition 3 shows that if this solution is smaller than $(1-b)\bar{\tau}$, then there must be a solution to equation (8) which is also smaller than $(1-b)\bar{\tau}$. Again, there is no guarantee that there exists a unique solution, but multiple solutions do not arise in examples. An interesting feature of this solution is that increasing the politician's bias will not necessarily make the optimal limit more stringent. This is because, while shifting up the curve $H(\ell) - H(\ell/(1+b))$, an increase in b also shifts up the curve $H(\ell/(1-b)) - H(\ell)$. The net effect is ambiguous.²⁰ Section 7 provides examples which illustrate that the stringency of the optimal limit can be both decreasing and increasing in bias.

When the solution is described by equation (8), it is no longer true that the probability an override occurs equals the probability the policy is equal to the limit. The former probability is $1 - H(\ell)$, while the latter is $H(\ell) - H(\ell/(1+b))$. Condition (8) implies that the latter probability

²⁰ The derivative of the function implicitly defined by equation (8) is given by

$$\frac{d\ell}{db} = \frac{h\left(\frac{\ell}{1-b}\right)\frac{\ell}{(1-b)^2} - h\left(\frac{\ell}{1+b}\right)\frac{\ell}{(1+b)^2}}{2h(\ell) - \frac{h\left(\frac{\ell}{1-b}\right)}{1-b} - \frac{h\left(\frac{\ell}{1+b}\right)}{1+b}}.$$

Assuming that $H(\ell) - H(\ell/(1+b))$ intersects $H(\ell/(1-b)) - H(\ell)$ from below, the denominator in this expression will be positive. However, the sign of the numerator is ambiguous.

is less than the former. Thus, the general result that is true for all bias levels is that under an optimal limit, the probability there is an override is at least as big as the probability the policy is set at the limit. While not as sharp as equality, this is still useful. For example, it is not satisfied in any of the cases studied in Section 3.²¹

It would be nice to know exactly when each of the two possibilities described in Proposition 3 arise. The possibility that both equations (7) and (8) have solutions in the relevant ranges make it difficult to fully characterize when each of the two cases will arise.²² However, our next Proposition provides a sufficient condition for the solution to adhere to equation (8).

Proposition 4 *If the politician's bias b is smaller than $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$ and if*

$$1 - H((1 - b)\bar{\tau}) < H((1 - b)\bar{\tau}) - H\left(\frac{\bar{\tau}}{1 + 2b}\right), \quad (9)$$

the optimal limit is smaller than $(1 - b)\bar{\tau}$ and solves equation (8).

The proof of this Proposition establishes that, when the condition holds, there cannot be a solution to (7) on the interval $[(1 - b)\bar{\tau}, \bar{\tau}]$. The condition requires that it is less likely that the citizen's preferred tax lies in the interval $[(1 - b)\bar{\tau}, \bar{\tau}]$ than in the interval $[\bar{\tau}/(1 + 2b), (1 - b)\bar{\tau}]$.²³ The latter interval is non-empty for values of b in the interval $(0, 1/2)$. Note also that the former interval has length $b\bar{\tau}$, while the latter has length $b\bar{\tau}(1 - 2b)/(1 + 2b)$. Thus, the condition requires that the distribution has less mass in its tails and will not be satisfied for the uniform distribution. Intuitively, the condition reveals that when it is relatively unlikely that the citizen's preferred tax is close to the top of the distribution, it is better to set a lower limit that will bind in more likely scenarios and allow the politician to set his preferred tax (through the override) when the citizen's preferred tax ends up being very large.

²¹ The statistics reported in Section 3 would capture the relevant probabilities in a stationary world in which each locality was ex ante identical but experienced a different shock. Reality is complicated by properties of the policies which result in districts facing different limits, dynamic features which may alter incentives, and the heterogeneity of localities. Further research is necessary to incorporate these considerations.

²² Since equations (7) and (8) coincide when ℓ is greater than $(1 - b)\bar{\tau}$, this possibility can only occur when equation (8) has multiple solutions.

²³ For distributions for which equation (8) has only one solution over the range $[\tau_m, \bar{\tau}]$ (and as a result, only one of equations (7) and (8) has a solution in the relevant range), like the examples described in Section 7, a more permissive sufficient condition is available. This condition requires that it is less likely that the citizen's preferred tax lies in the interval $[(1 - b)\bar{\tau}, \bar{\tau}]$ than in the interval $[(1 - b)\bar{\tau}/(1 + b), (1 - b)\bar{\tau}]$. Under this condition, decreasing the limit marginally from $(1 - b)\bar{\tau}$ will improve the citizen's welfare. The proof of Proposition 3 shows that in this case there must be a solution to equation (8) at a limit less than $(1 - b)\bar{\tau}$.

5 The impact of the override provision

To understand the impact of allowing overrides, this section characterizes the optimal limit without overrides and compares the findings with those of the previous section. With no overrides and limit ℓ , the politician will choose a tax ℓ if this is smaller than his preferred tax $(1+b)\tau$. Otherwise, he will choose his preferred tax. Thus his policy choice will be $\min\{\ell, (1+b)\tau\}$. As a result, the citizen's expected welfare will be given by

$$V_n(\ell) = - \int_{\underline{\tau}}^{\bar{\tau}} |\min\{\ell, (1+b)\tau\} - \tau| h(\tau) d\tau. \quad (10)$$

The designer's problem without overrides is then to choose a limit from the interval $[\underline{\tau}, \bar{\tau}]$ to maximize this function.

To understand the relationship between the problems with and without overrides, note that $V_n(\ell)$ will equal $V(\ell)$ if the limit ℓ is at least as big as $(1-b)\bar{\tau}$. If this condition is satisfied, while overrides are used in equilibrium, they do not benefit the citizen, because the politician uses his agenda-setting power to leave the citizen with the same utility as he would get from the limit. It is only when the limit ℓ is smaller than $(1-b)\bar{\tau}$ that overrides benefit the citizen and $V(\ell)$ exceeds $V_n(\ell)$. It follows that if the optimal limit with overrides is at least as big as $(1-b)\bar{\tau}$, then the optimal limit without overrides is equal to that with.

Lemma 1 tells us that the optimal limit with overrides is always at least as big as τ_m . Accordingly, if the politician's bias exceeds $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$, the optimal limit with overrides will exceed $(1-b)\bar{\tau}$ and there is no difference between the optimal limit with and without overrides. The optimal limit without overrides is then described by Propositions 1 and 2. This leaves the case when the politician's bias is smaller than $(\bar{\tau} - \underline{\tau})/2\bar{\tau}$. Propositions 3 and 4 tell us that the optimal limit with overrides can be smaller than $(1-b)\bar{\tau}$, in which case the optimal limit without overrides will not equal that with overrides. In this case, the optimal limit without overrides must satisfy equation (7). Intuitively, the calculus underlying the optimal limit with overrides when ℓ is at least as big as $(1-b)\bar{\tau}$ described above still applies directly to the case without overrides. Thus, we have:

Proposition 5 *Without overrides, if the politician's bias b is greater than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$, the optimal limit is τ_m . If the politician's bias is less than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$, the optimal limit solves equation (7).*

Comparing Propositions 3 and 5, the optimal limit with overrides differs from that without when the former is smaller than $(1-b)\bar{\tau}$ and solves equation (8). Proposition 4 provides us with

a sufficient condition for this to be the case. In this scenario, the optimal limit with overrides is *more stringent* than that without. To see why, consider that at any solution to equation (8) that is less than $(1 - b)\bar{\tau}$, equation (7) will not be satisfied since $1 - H(\ell)$ will exceed $H(\ell) - H(\ell/(1 + b))$. By the logic of the proof to Proposition 3, equation (7) must have a solution at a higher tax level. This finding makes good sense intuitively, since imposing a more stringent limit will be less costly when it can be overridden and the politician's override proposal provides the citizen with more than his utility from the limit.

We can draw on the literature on the delegation problem to help us understand these findings. Consider first Proposition 5. The literature tells us that the optimal limit without overrides will maximize the citizen's expected welfare given what can be inferred about the state of nature from the fact that the politician is bound by the limit.²⁴ In the large bias case, the politician setting taxes at the limit provides no information about the state, since the politician would do this regardless. Accordingly, the limit is set to maximize the citizen's expected welfare with no additional information on the state. Given the assumptions on preferences, this is the median of the distribution on $[\underline{\tau}, \bar{\tau}]$. In the small bias case, the politician setting taxes at the limit provides the information that $(1 + b)\tau$ is greater than ℓ . The optimal limit is set to maximize the citizen's expected welfare conditional on this information. Given the assumptions on preferences, this is the median of the truncated distribution on $[\ell/(1 + b), \bar{\tau}]$

Turning to Propositions 1 and 2, the logic underlying Proposition 5 continues to apply despite the fact that there are overrides because the agenda-setting power of the politician leaves the citizen with the same utility as he would get with the limit. The case covered by Proposition 3 is more interesting. In this case, when the optimal limit is smaller than $(1 - b)\bar{\tau}$, the politician may not set taxes such that the citizen receives his limit utility when an override occurs. Thus, the fact that the politician sets taxes such that the citizen receives his limit utility provides more information about the state. Specifically, $(1 + b)\tau$ is greater than ℓ but also exceeds $2\tau - \ell$. Again, the optimal limit is at the tax level that, conditional on this information, the citizen would prefer, which is the median of the truncated distribution on $[\ell/(1 + b), \ell/(1 - b)]$. The same intuition that explains the optimal limit without overrides, therefore explains the optimal limit with overrides, even when they differ.

²⁴ Holmstrom (1984) shows this for an example with quadratic distance preferences and Kartik, Van Weelden, and Wolton (2017) make a similar intuitive description of the result for interval delegation sets.

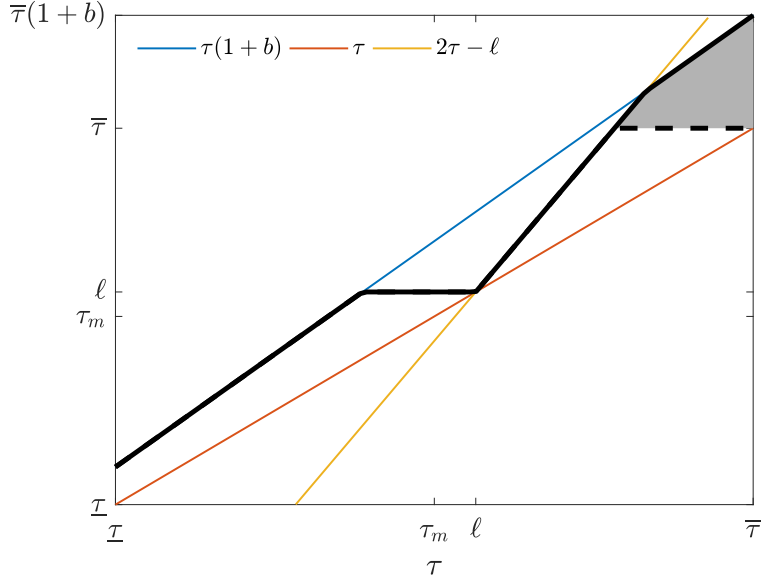


Figure 1: Welfare Gain of an Override Limit

6 The case for an override limit

This section shows that the institutional arrangement consisting of a limit and an override provision can always be strictly dominated by an arrangement that also specifies a limit on the policy that can be proposed through an override. It further explores how this option alters the designer's problem and characterizes the optimal system of limits which now consists of a regular limit and an override limit. Before getting into the details, we note that it can be demonstrated very easily that an override limit is beneficial. Figure 1 illustrates the case in which the optimal limit with overrides solves equation (8) as described in Proposition 3. The range of values for the citizen's preferred tax τ is measured on the horizontal axis and the three upward sloping lines are $(1+b)\tau$, τ , and $2\tau - \ell$ respectively. The heavy-set line describes the policy outcome under the optimal limit with overrides. If an override limit of $\bar{\tau}$ were imposed, the policy outcome would follow the dashed line for τ realizations above $\bar{\tau}/(1+b)$. This would create a gain in surplus for the citizen equal to the shaded area. A similar figure can be drawn for the optimal limits described in Propositions 1 and 2.

6.1 Extending the model to allow an override limit

Assume the constitutional designer now specifies a regular limit ℓ_r from the interval $[\underline{\tau}, \bar{\tau}]$ and an override limit ℓ_o from the interval $[\ell_r, \bar{\tau}]$. The politician's proposed tax t must always be less than

ℓ_o . If it does not exceed ℓ_r , it is implemented. If it exceeds ℓ_r , an override election is held. If the citizen votes in favor, the proposal is implemented. If the citizen votes against, the politician chooses another tax t' which respects the regular limit and this is implemented.

We now derive the policy implications of any given pair of limits (ℓ_r, ℓ_o) . As in the basic model, if the politician has to satisfy the regular limit ℓ_r , his policy choice will be $\min\{\ell_r, (1+b)\tau\}$. If τ is less than ℓ_r , the citizen will prefer this policy to any higher level and there is no point in the politician proposing to violate the regular limit. In this case, therefore, the implemented tax will be $\min\{\ell_r, (1+b)\tau\}$. If τ exceeds ℓ_r , the citizen will support taxation in excess of the limit and the optimal policy proposal for the politician solves the problem

$$\begin{aligned} \max_{t \leq \ell_o} & -|t - (1+b)\tau| \\ \text{s.t.} & -|t - \tau| \geq -|\ell_r - \tau|. \end{aligned} \quad (11)$$

This is the same as (1), except the politician must respect the override limit. The maximum tax the citizen will support is $2\tau - \ell_r$ and the politician will propose this if it is smaller than his preferred tax and the override limit. Otherwise, he will choose the minimum of his preferred tax and the override limit. Thus, the policy implemented if τ exceeds ℓ_r is $\min\{2\tau - \ell_r, (1+b)\tau, \ell_o\}$. It follows that with limit pair (ℓ_r, ℓ_o) , the citizen's expected welfare will be given by

$$V(\ell_r, \ell_o) = - \int_{\underline{\tau}}^{\ell_r} [\min\{\ell_r, (1+b)\tau\} - \tau] h(\tau) d\tau - \int_{\ell_r}^{\bar{\tau}} |\min\{2\tau - \ell_r, (1+b)\tau, \ell_o\} - \tau| h(\tau) d\tau. \quad (12)$$

The designer's problem is now to choose a pair of limits (ℓ_r, ℓ_o) to maximize this function.

Two conditions again shape the nature of the solution. As in the basic model, the first is whether the regular limit is greater or less than $(1+b)\underline{\tau}$. This determines the policy chosen if τ is less than ℓ_r in exactly the same way as before. The second condition is whether the regular limit ℓ_r is greater or less than $(1-b)\ell_o/(1+b)$, which determines what happens if τ exceeds ℓ_r . Here things differ from the basic model. If ℓ_r exceeds $(1-b)\ell_o/(1+b)$, then

$$\min\{2\tau - \ell_r, (1+b)\tau, \ell_o\} = \begin{cases} 2\tau - \ell_r & \text{if } \tau \in [\ell_r, \frac{\ell_r + \ell_o}{2}] \\ \ell_o & \text{if } \tau \in [\frac{\ell_r + \ell_o}{2}, \bar{\tau}] \end{cases}, \quad (13)$$

while if ℓ_r is less than $(1-b)\ell_o/(1+b)$, then

$$\min\{2\tau - \ell_r, (1+b)\tau, \ell_o\} = \begin{cases} 2\tau - \ell_r & \text{if } \tau \in [\ell_r, \frac{\ell_r}{1-b}] \\ (1+b)\tau & \text{if } \tau \in [\frac{\ell_r}{1-b}, \frac{\ell_o}{1+b}] \\ \ell_o & \text{if } \tau \in [\frac{\ell_o}{1+b}, \bar{\tau}] \end{cases} \quad (14)$$

6.2 The optimal system with large politician bias

If $(1+b)\underline{\tau}$ exceeds $\bar{\tau}$, any regular limit will be less than $(1+b)\underline{\tau}$ and the first condition is resolved. Furthermore, if $\underline{\tau}$ exceeds $(1-b)\bar{\tau}/(1+b)$, then for any pair of limits (ℓ_r, ℓ_o) , ℓ_r is at least as big as $(1-b)\ell_o/(1+b)$ and the second condition is resolved. Both inequalities will be satisfied if b is greater than $(\bar{\tau} - \underline{\tau})/\underline{\tau}$. Therefore, under this condition, we have that

$$V(\ell_r, \ell_o) = \int_{\underline{\tau}}^{\ell_r} [\tau - \ell_r] h(\tau) d\tau + \int_{\ell_r}^{\frac{\ell_r + \ell_o}{2}} [\ell_r - \tau] h(\tau) d\tau + \int_{\frac{\ell_r + \ell_o}{2}}^{\ell_o} [\tau - \ell_o] h(\tau) d\tau + \int_{\ell_o}^{\bar{\tau}} [\ell_o - \tau] h(\tau) d\tau. \quad (15)$$

Inspecting this objective function, we see that the regular limit effectively binds when the citizen's preferred tax is in the range $[\underline{\tau}, \frac{\ell_r + \ell_o}{2}]$ and the override limit binds when τ is in the range $[\frac{\ell_r + \ell_o}{2}, \bar{\tau}]$. By now familiar reasoning, the regular limit will be optimally set equal to the median of the truncated distribution on $[\underline{\tau}, \frac{\ell_r + \ell_o}{2}]$ and the override limit will be optimally set equal to the median of the truncated distribution on $[\frac{\ell_r + \ell_o}{2}, \bar{\tau}]$. Thus, we have:

Proposition 6 *With an override limit, if the politician's bias b exceeds $(\bar{\tau} - \underline{\tau})/\underline{\tau}$, the optimal system of limits (ℓ_r, ℓ_o) satisfies the equations*

$$H(\ell_r) = H\left(\frac{\ell_r + \ell_o}{2}\right) - H(\ell_r) \text{ and } H(\ell_o) - H\left(\frac{\ell_r + \ell_o}{2}\right) = 1 - H(\ell_o). \quad (16)$$

Given the symmetry assumption, one solution to system (16) is $H(\ell_r) = 1/4$ and $H(\ell_o) = 3/4$, in which case $(\ell_r + \ell_o)/2 = \tau_m$. In this case, the regular limit is at the median of the lower half of the distribution and the override limit is at the median of the upper half. This will be the unique solution if the density function $h(\tau)$ is relatively flat (for example, if for all τ' and τ , $h(\tau')/h(\tau) \leq 2$). As shown in the On-line Appendix, without such a condition, there may be multiple solutions and the solution which maximizes the objective function is not necessarily this one. However, in any solution, it is clear that the regular limit is strictly less than τ_m and the override limit is strictly larger than τ_m . Given Proposition 1, it follows that having an override limit makes overrides more likely. This seems a natural finding given that an override limit restricts the politician's ability to exploit his agenda-setting power.

In the optimal system described in Proposition 6, the general implication of optimality with overrides remains true for the regular limit. The probability an override occurs must exceed the probability the policy is equal to the regular limit. The former probability is $1 - H(\ell_r)$, while the latter is $H(\ell_r)$. The first equation of system (16) implies the former exceeds the latter. In fact, a sharper implication is true: the probability the policy is equal to the regular limit must equal the probability an override occurs and the policy is less than the override limit. This is because the latter probability is exactly $H((\ell_r + \ell_o)/2) - H(\ell_r)$.

6.3 The optimal system with small politician bias

If the politician's bias is below $(\bar{\tau} - \underline{\tau})/8\underline{\tau}$, the system of limits described in Proposition 6 will not be optimal. In this case, equation (16) implies that ℓ_r exceeds $(1+b)\underline{\tau}$ and thus the politician chooses his preferred tax at low realizations of τ . However, the expression for the citizen's welfare in equation (15) assumes that the politician chooses ℓ_r at low realizations of τ . The calculus giving rise to equations (16) is therefore no longer valid. In this situation, there are two possible solutions, in both of which ℓ_r exceeds $(1+b)\underline{\tau}$. These two solutions are distinguished by whether ℓ_r is larger or smaller than $(1-b)\ell_o/(1+b)$. In the former case, the citizen's welfare is given by

$$\begin{aligned} V(\ell_r, \ell_o) = & - \int_{\underline{\tau}}^{\frac{\ell_r}{1+b}} b\tau h(\tau) d\tau + \int_{\frac{\ell_r}{1+b}}^{\ell_r} [\tau - \ell_r] h(\tau) d\tau + \int_{\ell_r}^{\frac{\ell_r + \ell_o}{2}} [\ell_r - \tau] h(\tau) d\tau \\ & + \int_{\frac{\ell_r + \ell_o}{2}}^{\ell_o} [\tau - \ell_o] h(\tau) d\tau + \int_{\ell_o}^{\bar{\tau}} [\ell_o - \tau] h(\tau) d\tau. \end{aligned} \quad (17)$$

Note that the regular limit is effectively binding in the range $[\frac{\ell_r}{1+b}, \frac{\ell_r + \ell_o}{2}]$, while the override limit is binding in the range $[\frac{\ell_r + \ell_o}{2}, \bar{\tau}]$. The optimal regular limit is therefore at the median of the truncated distribution on $[\frac{\ell_r}{1+b}, \frac{\ell_r + \ell_o}{2}]$ and the optimal override limit at the median of the truncated distribution on $[\frac{\ell_r + \ell_o}{2}, \bar{\tau}]$. In the latter case, welfare is given by

$$\begin{aligned} V(\ell_r, \ell_o) = & - \int_{\underline{\tau}}^{\frac{\ell_r}{1+b}} b\tau h(\tau) d\tau + \int_{\frac{\ell_r}{1+b}}^{\ell_r} [\tau - \ell_r] h(\tau) d\tau + \int_{\ell_r}^{\frac{\ell_r}{1-b}} [\ell_r - \tau] h(\tau) d\tau \\ & - \int_{\frac{\ell_r}{1-b}}^{\frac{\ell_o}{1-b}} b\tau h(\tau) d\tau + \int_{\frac{\ell_o}{1-b}}^{\ell_o} [\tau - \ell_o] h(\tau) d\tau + \int_{\ell_o}^{\bar{\tau}} [\ell_o - \tau] h(\tau) d\tau. \end{aligned} \quad (18)$$

The regular limit effectively binds in the range $[\frac{\ell_r}{1+b}, \frac{\ell_r}{1-b}]$ and the override limit binds in the range $[\frac{\ell_o}{1+b}, \bar{\tau}]$. The optimal regular limit is therefore at the median of the truncated distribution on $[\frac{\ell_r}{1+b}, \frac{\ell_r}{1-b}]$ and the optimal override limit at the median of the truncated distribution on $[\frac{\ell_o}{1+b}, \bar{\tau}]$. Thus, we have:

Proposition 7 *With an override limit, if the politician's bias b is below $(\bar{\tau} - \underline{\tau})/8\underline{\tau}$, the optimal*

system of limits (ℓ_r, ℓ_o) is **either** such that $\ell_r \geq (1-b)\ell_o/(1+b)$ and satisfies the equations

$$H(\ell_r) - H\left(\frac{\ell_r}{1+b}\right) = H\left(\frac{\ell_r + \ell_o}{2}\right) - H(\ell_r) \text{ and } H(\ell_o) - H\left(\frac{\ell_r + \ell_o}{2}\right) = 1 - H(\ell_o), \quad (19)$$

or is such that $\ell_r < (1-b)\ell_o/(1+b)$ and satisfies the equations

$$H(\ell_r) - H\left(\frac{\ell_r}{1+b}\right) = H\left(\frac{\ell_r}{1-b}\right) - H(\ell_r) \text{ and } H(\ell_o) - H\left(\frac{\ell_o}{1+b}\right) = 1 - H(\ell_o). \quad (20)$$

The two solutions identified in the proposition are analogous to the two cases identified in Proposition 3.²⁵ Whichever case arises, ℓ_r is bounded above by the optimal limit with overrides and ℓ_o is bounded below by the optimal limit without overrides. The former means that overrides are at least as likely with an override limit as without. When the optimal system satisfies solution (19), both limits are decreasing in b . However, when it satisfies solution (20), ℓ_o is decreasing in b but ℓ_r need not be. This follows immediately from the fact that in this case, the equation defining ℓ_r in (20) is identical to (8), while the equation defining ℓ_o is identical to (7). Since $\ell_r < (1-b)\ell_o/(1+b)$ implies that $\ell_r < (1-b)\bar{\tau}$, when the optimal system satisfies solution (20), ℓ_r will coincide with the optimal limit with overrides and ℓ_o will coincide with the optimal limit without overrides in cases without multiple solutions.

In both the solutions described in Proposition 7, the probability an override occurs must exceed the probability the policy is equal to the regular limit. These two probabilities remain $1 - H(\ell_r)$ and $H(\ell_r)$, respectively. The first equation of each system implies the former exceeds the latter. In the first solution, it also remains true that the probability the policy is equal to the regular limit must equal the probability an override occurs and the policy is less than the override limit. However, in the second solution, the former probability is lower than the latter.

6.4 The optimal system with medium politician bias

Propositions 6 and 7 cover the cases in which bias b is below $(\bar{\tau} - \underline{\tau})/8\underline{\tau}$ and above $(\bar{\tau} - \underline{\tau})/\underline{\tau}$. This leaves a large middle range in between. In this range, the optimal system of limits will either satisfy (16), (19), or (20). This can be established using the arguments found in the proof of Proposition 7. At higher ranges, the optimal system will satisfy (16) and in lower ranges either (19) or (20). Due to the possibility of multiple solutions to (16), it is difficult to characterize a critical value of b which delineates the two ranges. The next section shows how all this plays

²⁵ The case in which the optimal system satisfies (19) is analogous to the optimal limit with overrides satisfying equation (7), while the case in which the optimal system satisfies (20) is analogous to the optimal limit with overrides satisfying equation (8).

out in specific examples. It can be shown in general that the optimal regular limit ℓ_r is bounded above by the optimal limit with overrides and the optimal override limit ℓ_o is bounded below by the optimal limit without overrides. The On-line Appendix provides the details.

7 Examples

This section presents two examples involving different distributions of the citizen's preferred tax. These examples are used to illustrate how the optimal limit with overrides depends on the politician's bias. They are also used to shed light on how the optimal limit with overrides differs from that without and on what the optimal system looks like if an override limit is available. Throughout this section, the optimal limit with overrides is denoted ℓ , the optimal limit without overrides ℓ_n , and the optimal regular and override limits ℓ_r and ℓ_o .

7.1 Uniform distribution

Suppose that the distribution of the citizen's preferred tax is uniform; i.e., $H(\tau)$ equals $(\tau - \underline{\tau}) / (\bar{\tau} - \underline{\tau})$. Then, when the politician's bias b is less than $(\bar{\tau} - \underline{\tau}) / 2\underline{\tau}$, the optimal limits with and without overrides are

$$\ell = \ell_n = \bar{\tau} \left(\frac{1+b}{1+2b} \right). \quad (21)$$

To see this, note first that equation (8) has no solution in the uniform case if $b > 0$.²⁶ Propositions 2 and 3 therefore imply that the optimal limit with overrides satisfies equation (7) and solving this yields (21). Proposition 5 tells us that the optimal limit without overrides is also equal to (21). Intuitively, while overrides occur under the optimal limit, the politician's proposal is always such as to leave the citizen with his limit utility.

Equation (21) implies that as b approaches 0, the limit converges to $\bar{\tau}$ and so the politician becomes completely unconstrained as bias vanishes. Without overrides, this is natural, because the politician is becoming a perfect agent for the citizen and there is little gain from constraining him. With overrides, the limit is irrelevant when the politician is a perfect agent of the citizen and therefore it is not clear to what point the limit will converge. Thus, the result is more surprising. Note also that the limit gets progressively tighter as we increase b , until the point at which b equals $(\bar{\tau} - \underline{\tau}) / 2\underline{\tau}$ and the limit equals the median preferred tax τ_m .

²⁶ For equation (8) to be satisfied $H(\ell) - H(\ell/(1+b))$ must equal $H(\ell/(1-b)) - H(\ell)$. In the uniform case, this is not possible since $\ell - \ell/(1+b)$ cannot be equal to $\ell/(1-b) - \ell$, unless $b = 0$.

When an override limit is available, the optimal system of limits is

$$(\ell_r, \ell_o) = \begin{cases} \left(\frac{\bar{\tau} + 3\underline{\tau}}{4}, \frac{3\bar{\tau} + \underline{\tau}}{4} \right) & \text{if } b \geq (\bar{\tau} - \underline{\tau}) / 4\underline{\tau} \\ \left(\bar{\tau} \left(\frac{1+b}{1+4b} \right), \bar{\tau} \left(\frac{1+3b}{1+4b} \right) \right) & \text{if } b \in (0, (\bar{\tau} - \underline{\tau}) / 4\underline{\tau}) \end{cases}. \quad (22)$$

In the higher bias range, the optimal system is the solution to system (16). In the lower range, it corresponds to system (19). In this case, both the regular limit and the override limit become more permissive as the politician becomes less biased and converge to $\bar{\tau}$ as bias vanishes. In both ranges, ℓ_r is strictly below ℓ and ℓ_o is strictly greater than ℓ_n . The former result means overrides are always more likely with an override limit.

It is possible to analyze the impact of changing the distribution of preferred tax levels by considering a parameterization in which $(\underline{\tau}, \bar{\tau})$ equals $(\eta - z, \eta + z)$. Increasing z implements a mean preserving spread since it holds constant the citizen's expected preferred level but increases the dispersion. Equations (21) and (22) imply that optimal limits become more permissive with a mean preserving spread. Related comparative static findings are highlighted in a number of papers in the literature on the delegation problem. Indeed, Huber and Shipan (2006) refer to the idea that the optimal permissible set of actions for the agent is increased when the principal faces more uncertainty as the *Uncertainty Principle*.

7.2 Tent distribution

Suppose that the distribution of the citizen's preferred tax is a tent distribution; i.e.,

$$H(\tau) = \begin{cases} \frac{(\tau - \underline{\tau})^2}{2(\tau_m - \underline{\tau})^2} & \text{if } \tau \in [\underline{\tau}, \tau_m] \\ \frac{1}{2} + \frac{(\tau - \tau_m)[2\bar{\tau} - (\tau + \tau_m)]}{2(\tau_m - \underline{\tau})^2} & \text{if } \tau \in [\tau_m, \bar{\tau}] \end{cases}. \quad (23)$$

The density associated with this distribution rises linearly from 0 to $1/(\tau_m - \underline{\tau})$ over the interval $[\underline{\tau}, \tau_m]$ and comes back down the other side. Despite the simplicity of this distribution, solving for the optimal limits in this case is reasonably involved, so we use numerical methods.

Panel A of Figure 2 depicts the optimal limits when $(\underline{\tau}, \bar{\tau})$ equals (0.1, 0.3). The optimal limit with overrides is the dotted yellow curve, the higher of the two curves that take on the value 0.2 when b is equal to 0. The optimal limit without overrides is the dashed purple curve that diverges from the dotted curve when b is equal to 0.3. At low levels of bias, the two limits diverge substantially: ℓ_n converges to $\bar{\tau}$ as b approaches zero while ℓ does not. This difference reflects the fact that ℓ_n is determined by equation (7), while ℓ is determined by equation (8). Indeed, while ℓ_n

is relaxed as bias falls, ℓ becomes more stringent over a good part of its range, converging to τ_m as b approaches zero. This illustrates the theoretical ambiguity noted earlier when equation (8) determines the optimal limit. The result contrasts with findings in the literature on the delegation problem which show that the optimal permissible set of actions for the agent is increased when bias is reduced (the so-called *Ally Principle*).

When an override limit is possible, the optimal regular and override limits are illustrated by the solid blue and solid orange lines in Panel A of Figure 2. For very low values of b ($b \leq 0.09$), the optimal system solves (20). Since both equation (8) and equation (7) have unique solutions, ℓ_r is equal to ℓ and ℓ_o is equal to ℓ_n . Given that ℓ is increasing in b in this range, so is ℓ_r . For intermediate values of b ($b \in (0.09, 0.68]$) the optimal system solves (19). In this range, ℓ_r is less than ℓ and ℓ_o exceeds ℓ_n . Now, both ℓ_r and ℓ_o are decreasing in b . For higher values of b ($b > 0.68$) the optimal system solves (16) and ℓ_r is again less than ℓ and ℓ_o exceeds ℓ_n . The two limits are independent of b .

Panel B of Figure 2 illustrates the optimal limits for the case in which $(\tau, \bar{\tau})$ equals $(0, 0.4)$. This represents a more spread distribution than Panel A. While the relationship between all the various limits is basically the same as in Panel A, there are a couple of interesting points to note. First, ℓ and ℓ_r no longer become more stringent as bias falls. In this example, this feature turns out to depend on the spread of the distribution. A greater spread restores the Ally Principle. Nonetheless, there remain clear differences between the behavior of ℓ and ℓ_n in response to bias reductions: ℓ_n is relaxed at an increasing rate, while ℓ is relaxed at a decreasing rate. Second, all four limits increase relative to their values in Panel A, consistent with the Uncertainty Principle.

8 Conclusion

Fiscal limits are commonplace in state and local government in the US. It is therefore important to understand how they should be designed. The contract theory literature on the delegation problem is a natural place to look for guidance. However, this literature ignores a key feature of how limits are structured in reality: namely, the possibility of overrides. Overrides are not only possible in principle, but are widely used in practice. Accordingly, this paper has studied the optimal design of fiscal limits in a model that extends the standard framework of the delegation problem to incorporate the possibility of overrides.

The analysis of the model sheds light on how the optimal limit depends on the possibility of overrides and the level of bias towards larger government in the political system. When the level of

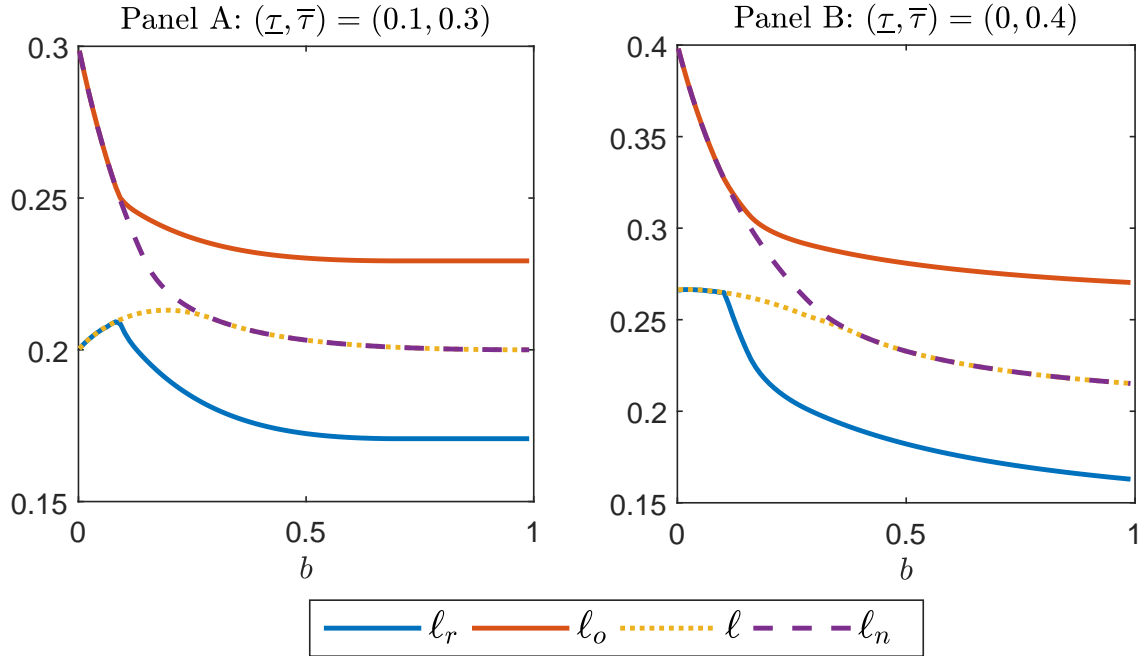


Figure 2: Optimal Limits for the Tent Distribution

bias is high, the optimal limit is the same with or without overrides and equals the median level of spending or taxation the representative citizen expects to desire. For smaller levels of bias, more permissive limits are optimal and the optimal limit with overrides can differ from that without. A necessary condition for a difference to emerge is that override proposals leave citizens with some benefits relative to their payoffs from the limit level of the policy. When there is a difference, optimal limits with overrides are more stringent than those without. In addition, the way in which the optimal limit responds to changes in bias is impacted. Most notably, with overrides, it is not necessarily the case that the optimal limit becomes more permissive as the level of bias falls. This will depend on the distribution of preferred tax levels.

The analysis also reveals how the institutional arrangement consisting of a limit and an override provision can be improved upon. Imposing a limit that caps the proposal the politician can make when using an override will always be optimal. Intuitively, such an override limit restricts the ability of the politician to exploit his agenda-setting power. Such a reform is conceptually simple and there would seem no obvious practical difficulties preventing its introduction. Indeed, as pointed out in Section 3, Massachusetts does limit override proposals. Of course, with such a reform, the constitutional designer would have to select two limits: a regular limit and an override

limit. The analysis provides guidance as to how such a system should be designed. In general, the optimal regular limit is at least as stringent as the optimal limit with overrides and the optimal override limit is at least as permissive as the optimal limit without overrides.

There is considerable scope for further work on the practically important and theoretically interesting problem of designing fiscal limits. Extensions of the model readily suggest themselves. To understand the generality of the results found here, it may be worth considering more general preferences for the citizen and politician. Similarly, relaxing the assumptions on the distribution of preferred tax levels and introducing richer uncertainty in the degree of politician bias may prove instructive. Recognizing that it is costly to hold an override election might yield interesting results. Finally, further potentially useful reforms might be identified by formulating and solving a mechanism design version of the problem.

In addition, there are interesting questions that a static model like that presented here cannot answer. As we pointed out in Section 3, there is lots of variation in practice in how limits evolve from one period to the next. This is particularly the case when overrides are used. Shedding light on how next period's limit should depend on what happens this period is an important and challenging problem for further research.

Considering optimal limits in a legislative setting would also be interesting. In such a setting, taxation is determined by the collective decisions of legislators rather than the decision of a single politician. It would be natural to consider override provisions which allow the limit to be overridden by a super-majority of legislators rather than by direct appeal to the citizens. Continually undertaking referenda is likely to prove administratively costly and it may be that the same function can be achieved by appropriate choice of super-majority override.

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9 Appendix

9.1 Proof of Proposition 1

We need to show that $V(\tau_m)$ exceeds $V(\ell)$ for any limit ℓ in the range $[\underline{\tau}, \tau_m)$ or $(\tau_m, \bar{\tau}]$. Since b exceeds $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$, ℓ is less than $(1+b)\underline{\tau}$ for any limit ℓ in the range $[\underline{\tau}, \tau_m]$ and ℓ exceeds $(1-b)\bar{\tau}$ for any limit ℓ in the range $[\tau_m, \bar{\tau}]$.

Consider a limit ℓ in the range $[\underline{\tau}, \tau_m)$. If ℓ exceeds $(1-b)\bar{\tau}$, we know from the argument in the text that $V(\tau_m)$ exceeds $V(\ell)$, so suppose that ℓ is less than $(1-b)\bar{\tau}$. In this case,

$$V(\ell) = \int_{\underline{\tau}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} [\ell - \tau] h(\tau) d\tau - \int_{\frac{\ell}{1-b}}^{\bar{\tau}} b\tau h(\tau) d\tau.$$

The impact of marginally increasing ℓ is

$$\begin{aligned} \int_{\ell}^{\frac{\ell}{1-b}} h(\tau) d\tau - \int_{\underline{\tau}}^{\ell} h(\tau) d\tau &= H\left(\frac{\ell}{1-b}\right) - 2H(\ell) \\ &> H\left(\frac{\ell}{1-b}\right) - H(\ell) - \frac{1}{2} \\ &= \int_{\ell}^{\tau_m} h(\tau) d\tau - \int_{\frac{\ell}{1-b}}^{\bar{\tau}} h(\tau) d\tau. \end{aligned}$$

We claim that

$$\int_{\ell}^{\tau_m} h(\tau) d\tau \geq \int_{\frac{\ell}{1-b}}^{\bar{\tau}} h(\tau) d\tau.$$

Given the assumed properties of h , a sufficient condition for this is that

$$\tau_m - \ell \geq \bar{\tau} - \frac{\ell}{1-b} \Leftrightarrow \frac{b\ell}{1-b} > \bar{\tau} - \tau_m.$$

But we know that

$$\frac{b\ell}{1-b} \geq \frac{b\underline{\tau}}{1-b} \geq \frac{(\bar{\tau} - \tau_m)}{1-b} > \bar{\tau} - \tau_m.$$

Now consider a limit ℓ in the range $(\tau_m, \bar{\tau}]$. If ℓ is less than $(1+b)\underline{\tau}$, we know from the argument in the text that $V(\tau_m)$ exceeds $V(\ell)$, so suppose that ℓ is greater than $(1+b)\underline{\tau}$. In this case,

$$V(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau.$$

The impact of marginally decreasing ℓ is

$$\begin{aligned} \int_{\frac{\ell}{1+b}}^{\ell} h(\tau) d\tau - \int_{\ell}^{\bar{\tau}} h(\tau) d\tau &= 2H(\ell) - 1 - H\left(\frac{\ell}{1+b}\right) \\ &> H(\ell) - \frac{1}{2} - H\left(\frac{\ell}{1+b}\right) \\ &= \int_{\tau_m}^{\ell} h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} h(\tau) d\tau. \end{aligned}$$

We claim that

$$\int_{\tau_m}^{\ell} h(\tau) d\tau \geq \int_{\underline{\tau}}^{\frac{\ell}{1+b}} h(\tau) d\tau.$$

Given the assumed properties of h , a sufficient condition for this is that

$$\ell - \tau_m \geq \frac{\ell}{1+b} - \underline{\tau} \Leftrightarrow \frac{b\ell}{1+b} > \tau_m - \underline{\tau}.$$

But we know that

$$\frac{b\ell}{1+b} \geq \frac{b\underline{\tau}}{1+b} \geq \frac{(\bar{\tau} - \tau_m)}{1+b} > \tau_m - \underline{\tau}.$$

■

9.2 Proof of Lemma 1

Proposition 1 implies that the result is true for b larger than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$. Thus, we just need to show that the result is true for b smaller than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$. Consider some limit $\ell < \tau_m$. We will show that marginally increasing ℓ will increase the citizen's payoff.

Suppose first that $\ell \geq (1-b)\bar{\tau}$. If $\ell \geq (1+b)\underline{\tau}$, then, from (2), we have that

$$V(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau.$$

Note that

$$V'(\ell) = - \int_{\frac{\ell}{1+b}}^{\ell} h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} h(\tau) d\tau = 1 + H\left(\frac{\ell}{1+b}\right) - 2H(\ell) > 0,$$

which implies that raising the limit slightly will increase the citizen's payoff. If $\ell < (1+b)\underline{\tau}$, then, from (2), we have that

$$V(\ell) = \int_{\underline{\tau}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau.$$

Note that

$$V'(\ell) = - \int_{\underline{\tau}}^{\ell} h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} h(\tau) d\tau = 1 - 2H(\ell) > 0,$$

which again implies that raising ℓ marginally benefits the citizen.

Now suppose that $\ell < (1-b)\bar{\tau}$. If $\ell \geq (1+b)\underline{\tau}$, then, from (2), we have that

$$V(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau - \int_{\frac{\ell}{1-b}}^{\bar{\tau}} b\tau h(\tau) d\tau.$$

Note that

$$V'(\ell) = - \int_{\frac{\ell}{1+b}}^{\ell} h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} h(\tau) d\tau.$$

Given that $\frac{\ell}{1+b} < \ell < \tau_m$ and that

$$\ell - \frac{\ell}{1+b} = \frac{b\ell}{1+b} < \frac{b\ell}{1-b} = \frac{\ell}{1-b} - \ell,$$

the assumption that h is symmetric and non-decreasing on $[\underline{\tau}, \tau_m]$ implies that

$$\int_{\ell}^{\frac{\ell}{1-b}} h(\tau) d\tau > \int_{\frac{\ell}{1+b}}^{\ell} h(\tau) d\tau.$$

To see this, note that for any $\tau \in [\frac{\ell}{1+b}, \ell]$ we can associate a unique $\tau' \in [\ell, \frac{\ell}{1-b}]$ (e.g., $\tau' = 2\ell - \tau$) which has a higher density. Thus, it must be the case that $V'(\ell) > 0$ which implies that raising the limit slightly will increase the citizen's payoff. If $\ell < (1+b)\underline{\tau}$, then, from (2), we have that

$$V(\ell) = \int_{\underline{\tau}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} [\ell - \tau] h(\tau) d\tau - \int_{\frac{\ell}{1-b}}^{\bar{\tau}} b\tau h(\tau) d\tau.$$

Note that

$$V'(\ell) = - \int_{\underline{\tau}}^{\ell} h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} h(\tau) d\tau.$$

Given that $\underline{\tau} < \ell < \tau_m$ and that

$$\ell - \underline{\tau} < \ell - \frac{\ell}{1+b} = \frac{b\ell}{1+b} < \frac{b\ell}{1-b} = \frac{\ell}{1-b} - \ell,$$

the assumption that h is symmetric and non-decreasing on $[\underline{\tau}, \tau_m]$ implies that

$$\int_{\ell}^{\frac{\ell}{1-b}} h(\tau) d\tau > \int_{\underline{\tau}}^{\ell} h(\tau) d\tau.$$

Again, to see this note that for any $\tau \in [\underline{\tau}, \ell]$ we can find a unique $\tau' \in [\ell, \frac{\ell}{1-b}]$ (e.g., $\tau' = 2\ell - \tau$) which has a higher density. Thus, $V'(\ell) > 0$ which again implies that raising ℓ marginally benefits the citizen. ■

9.3 Proof of Proposition 2

For limits $\ell \in [\tau_m, \bar{\tau}]$, we have that ℓ is greater than or equal to τ_m which, since b is less than $(\tau_m - \underline{\tau})/\underline{\tau}$, exceeds $(1+b)\underline{\tau}$. In addition, we have that ℓ is greater than or equal to τ_m which, since b exceeds $(\bar{\tau} - \tau_m)/\bar{\tau}$, exceeds $(1-b)\bar{\tau}$. Thus, for limits $\ell \in [\tau_m, \bar{\tau}]$, (2) implies that

$$V(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau.$$

This means that

$$V'(\ell) = - \int_{\frac{\ell}{1+b}}^{\ell} h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} h(\tau) d\tau = 1 - H(\ell) - \left(H(\ell) - H\left(\frac{\ell}{1+b}\right) \right).$$

It follows that at the optimal limit

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = 1 - H(\ell),$$

which is (7). To see that this equation has a solution, note that

$$H(\tau_m) - H\left(\frac{\tau_m}{1+b}\right) < 1 - H(\tau_m) = \frac{1}{2},$$

and that

$$H(\bar{\tau}) - H\left(\frac{\bar{\tau}}{1+b}\right) > 1 - H(\bar{\tau}) = 0.$$

Thus, by the *Intermediate Value Theorem*, there exists a solution to equation (7). \blacksquare

9.4 Proof of Proposition 3

For limits $\ell \in [\tau_m, \bar{\tau}]$, we have that ℓ is greater than or equal to τ_m which, since b is less than $(\bar{\tau} - \tau_m) / \bar{\tau}$, exceeds $(1+b)\underline{\tau}$. Moreover, since τ_m is less than $(1-b)\bar{\tau}$ which is less than $\bar{\tau}$, we can have that $\ell \geq (1-b)\bar{\tau}$. It follows from (2) that the citizen's welfare with limit $\ell \in [\tau_m, \bar{\tau}]$ is

$$V(\ell) = \begin{cases} \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\frac{\ell}{1-b}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau - \int_{\frac{\ell}{1-b}}^{\bar{\tau}} b\tau h(\tau) d\tau & \text{if } \ell < (1-b)\bar{\tau} \\ \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau & \text{if } \ell \geq (1-b)\bar{\tau} \end{cases}$$

Thus, the impact on welfare of a small increase in the limit is

$$V'(\ell) = \begin{cases} H(\frac{\ell}{1-b}) - H(\ell) - \left(H(\ell) - H(\frac{\ell}{1+b})\right) & \text{if } \ell < (1-b)\bar{\tau} \\ 1 - H(\ell) - \left(H(\ell) - H(\frac{\ell}{1+b})\right) & \text{if } \ell \geq (1-b)\bar{\tau} \end{cases}$$

It follows that the optimal limit is either such that $\ell \in [\tau_m, (1-b)\bar{\tau})$ and solves

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = H\left(\frac{\ell}{1-b}\right) - H(\ell),$$

or is such that $\ell \in [(1-b)\bar{\tau}, \bar{\tau}]$ and solves

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = 1 - H(\ell).$$

It is straightforward to show that at least one of these equations must have a solution in the relevant range. The assumption that b is less than $(\bar{\tau} - \bar{\tau}) / 2\bar{\tau}$ implies that b is less than $(\tau_m - \underline{\tau}) / \underline{\tau}$ which means that τ_m exceeds $(1+b)\underline{\tau}$. Thus,

$$1 - H(\tau_m) = \frac{1}{2} > H(\tau_m) - H\left(\frac{\tau_m}{1+b}\right).$$

Since

$$1 - H(\bar{\tau}) = 0 < H(\bar{\tau}) - H\left(\frac{\bar{\tau}}{1+b}\right),$$

there exists $\ell \in (\tau_m, \bar{\tau})$ such that

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = 1 - H(\ell).$$

Suppose that for all such ℓ it is the case that $\ell < (1-b)\bar{\tau}$, then it must be the case that

$$H((1-b)\bar{\tau}) - H\left(\frac{(1-b)\bar{\tau}}{1+b}\right) > 1 - H((1-b)\bar{\tau}),$$

or, equivalently, that

$$H((1-b)\bar{\tau}) - H\left(\frac{(1-b)\bar{\tau}}{1+b}\right) > H\left(\frac{(1-b)\bar{\tau}}{1-b}\right) - H((1-b)\bar{\tau}).$$

If

$$H\left(\frac{\tau_m}{1-b}\right) - H(\tau_m) > H(\tau_m) - H\left(\frac{\tau_m}{1+b}\right), \quad (24)$$

this implies that there exists $\ell \in [\tau_m, (1-b)\bar{\tau}]$ such that

$$H\left(\frac{\ell}{1-b}\right) - H(\ell) = H(\ell) - H\left(\frac{\ell}{1+b}\right).$$

It suffices, therefore, to prove that (24) holds. Note that symmetry implies that

$$H\left(\frac{\tau_m}{1+b}\right) = 1 - H\left(\tau_m + \frac{b\tau_m}{1+b}\right).$$

Moreover, we have that

$$\frac{\tau_m}{1-b} = \tau_m + \frac{b\tau_m}{1-b} > \tau_m + \frac{b\tau_m}{1+b}.$$

This means that

$$H\left(\frac{\tau_m}{1+b}\right) + H\left(\frac{\tau_m}{1-b}\right) > H\left(\frac{\tau_m}{1+b}\right) + H\left(\tau_m + \frac{b\tau_m}{1+b}\right) = 1 = 2H(\tau_m).$$

■

9.5 Proof of Proposition 4

Given Proposition 3, it suffices to show that, under the condition of the Proposition, equation (7) has no solution bigger than $(1-b)\bar{\tau}$. Suppose then that ℓ solves equation (7).

We first observe that ℓ must be less than or equal to $(1+b)\bar{\tau}/(1+2b)$. To see this, note first that, since h is decreasing over the interval $(\tau_m, \bar{\tau}]$, we have that

$$1 - H(\ell) = H(\bar{\tau}) - H(\ell) \leq h(\ell)(\bar{\tau} - \ell)$$

and that

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) \geq h(\ell) \left[\ell - \frac{\ell}{1+b} \right] = h(\ell) \frac{\ell b}{(1+b)}.$$

As a result, we have that

$$\begin{aligned} 1 - H(\ell) &\leq h(\ell)(\bar{\tau} - \ell) \\ &\leq \left(\frac{H(\ell) - H\left(\frac{\ell}{1+b}\right)}{\frac{\ell b}{(1+b)}} \right) (\bar{\tau} - \ell) \\ &= \left(H(\ell) - H\left(\frac{\ell}{1+b}\right) \right) \frac{(\bar{\tau} - \ell)(1+b)}{\ell b}. \end{aligned}$$

Given that ℓ solves (7), it must be the case that $H(\ell) - H(\ell/(1+b)) = 1 - H(\ell)$. The above inequality then requires that

$$\frac{(\bar{\tau} - \ell)(1+b)}{\ell b} \geq 1,$$

which in turn implies that

$$\ell \leq \bar{\tau} \frac{1+b}{1+2b}.$$

We now show that ℓ must be less than $(1-b)\bar{\tau}$. Suppose not. Then, it must be the case that $\ell \in [(1-b)\bar{\tau}, (1+b)\bar{\tau}/(1+2b)]$. Since H is increasing and $\ell \geq (1-b)\bar{\tau}$, we have that

$$1 - H((1-b)\bar{\tau}) \geq 1 - H(\ell).$$

Moreover, since $\ell \leq (1+b)\bar{\tau}/(1+2b)$, we have that

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) \geq H((1-b)\bar{\tau}) - H\left(\frac{\bar{\tau}}{1+2b}\right).$$

The condition of the Proposition tells us that

$$H((1-b)\bar{\tau}) - H\left(\frac{\bar{\tau}}{1+2b}\right) > 1 - H((1-b)\bar{\tau}).$$

Combining these three inequalities reveals that

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) > 1 - H((1-b)\bar{\tau}) \geq 1 - H(\ell),$$

which contradicts the fact that ℓ solves equation (7). ■

9.6 Proof of Proposition 5

The result for the case in which the politician's bias exceeds $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$ follows from argument in the text. Thus, we just need to deal with the case in which the politician's bias is less than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$.

We begin by showing that the optimal limit is always at least as big as τ_m . We establish this by demonstrating that with any limit $\ell \in [\underline{\tau}, \tau_m)$, marginally increasing ℓ will increase the citizen's expected welfare. Suppose first that $\ell \geq (1+b)\underline{\tau}$, then, from (10), we have that the citizen's welfare is

$$V_n(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau.$$

Note that this is equal to $V(\ell)$ when $\ell \geq (1-b)\bar{\tau}$ and $\ell \geq (1+b)\bar{\tau}$. By the argument in the proof to Lemma 1, raising the limit slightly will increase the citizen's welfare. If $\ell < (1+b)\underline{\tau}$ then, from (10), we have that

$$V_n(\ell) = \int_{\underline{\tau}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau.$$

Note that this is equal to $V(\ell)$ when $\ell \geq (1-b)\bar{\tau}$ and $\ell < (1+b)\bar{\tau}$. Again, by the argument in the proof to Lemma 1, this implies that raising ℓ will benefit the citizen.

Now for limits $\ell \in [\tau_m, \bar{\tau}]$, we have that $\ell/(1+b)$ is greater than or equal to $\tau_m/(1+b)$ which, since b is less than $(\bar{\tau} - \underline{\tau})/2\underline{\tau}$, exceeds $\underline{\tau}$. As a result, from (10), we have that the citizen's welfare is

$$V_n(\ell) = \int_{\frac{\ell}{1+b}}^{\ell} [\tau - \ell] h(\tau) d\tau + \int_{\ell}^{\bar{\tau}} [\ell - \tau] h(\tau) d\tau - \int_{\underline{\tau}}^{\frac{\ell}{1+b}} b\tau h(\tau) d\tau.$$

Thus, the impact on welfare of a small increase in the limit is

$$V'_n(\ell) = 1 - H(\ell) - (H(\ell) - H(\frac{\ell}{1+b})).$$

It follows that the optimal limit will satisfy

$$H(\ell) - H(\frac{\ell}{1+b}) = 1 - H(\ell),$$

which is equation (7). By the argument in the proof of Proposition 2, this equation has a solution.

■