On-Line Appendix to Optimal Fiscal Limits with Overrides

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1 The override provision in practice

This section provides additional details for the tax limits in the 3 states described in the paper.

1.1 Massachusetts

In Massachusetts, Proposition $2\frac{1}{2}$ limits the total amount of property tax local municipalities can levy, which is the predominant source of local tax revenue. Municipalities are the only major type of local government and over 95% of their tax revenue comes from property taxes (Urban Institute-Brookings Institution Tax Policy Center 2016). When the limit was enacted through a popular vote in 1983, each government's tax limit was set at its 1982 tax level. In each subsequent year, the limit has been equal to the prior year's limit plus 2.5% and an additional adjustment for increases in property values due to new construction.

To exceed its limit, the governing body of a municipality must propose a dollar amount of additional revenue and its purpose, and then allow the electorate to vote. If the override proposal receives majority approval, the city or town can tax the additional amount. If it does not, the government must stay within the limit. Since each year's limit is based upon the previous year, a successful override relaxes the limit in all subsequent years.

In addition, Proposition $2\frac{1}{2}$ created an absolute maximum level of tax revenue equal to 2.5% of the total assessed fair market value in the municipality. All proposed overrides must abide by this limit. Bradbury and Ladd (1982) state that the Massachusetts Department of Revenue estimated that this limit would bind for approximately half of the state's municipalities in the first year. However, following the marked increase in property values in subsequent years, it bound less than expected. Nonetheless a few municipalities have reached this limit in recent decades.

Cities and towns have used the override to significantly increase their taxing authority. Our data come from the Massachusetts Department of Revenue. As reported in the paper, 49% of municipalities overrode their limit between 2000 and 2016. Between 1983, when the limit began, and 2016, 74% of municipalities passed at least one override to exceed their limit.

Overrides have had a large impact on the overall level of taxes in Massachusetts. Because successful overrides increase the limit for all subsequent years and the 2.5% growth rate is applied to them in the process, their effects cascade through the years. Thus we can calculate the effect of all overrides in a jurisdiction on their 2016 tax limit with $\sum_{t=1983}^{2016} override_t 1.025^{2016-t}$, where $override_t$ is the value of the override passed in year t and zero if there was none. Thus overrides approved between 1983 and 2016 increased the statewide total levy limit in 2016 by $943 million, relative to if none had ever been passed. This represents 6.2% of the total 2016 levy limit.

This understates the importance of overrides, since in addition to directly changing the levy limit, an override will also make future additions to the limit due to new construction larger. We lack sufficient data to calculate this for the full timespan, but for overrides passed between 1992 and 2016 this impact was approximately $117 million. Limited to only those cities and towns that passed at least one override, the taxes approved via override represent 12.5% of their total 2016 levy limit.

As reported in the paper, municipalities on average set taxes at their limit (but do not override it) 44% of the time between 2000 and 2016. This number includes municipalities who could not have raised their taxes without exceeding their limit. In most cases this does not mean that their tax revenue was exactly equal to the limit because they set taxes as a tax rate with a limited number of decimal places and so cannot perfectly target an amount of tax revenue. Since the model offers no clear way to interpret them, we do not separate out occasions when an override was proposed but rejected by the voter from occasions when no override was proposed; these may be included in the number.

It is worth noting that not all tax revenues are covered by this limit. Tax revenues for capital projects and to repay debt also require voter approval but are governed by different rules. These referenda are not included in these numbers although they are also widely used.

1.2 Ohio

Since 1911, Ohio has limited the ability of local governments to set property tax rates, and allowed voters to approve higher taxes through referenda. Each year, local governments, acting with only their own authority, typically cannot increase tax revenues at all relative to the previous year. When previous, limited-term, taxes expire the taxing ability of the local government decreases. Thus, in real terms, their taxing ability typically declines from year to year. To exceed any year's limit they must ask the voters to approve a new tax levy.

In Ohio, multiple kinds of jurisdictions, including school districts, counties, municipalities, and special districts, have the authority to levy property taxes. Collectively, all coincident taxing jurisdictions can levy a property tax at a rate of 1% of the assessed property value (which is in turn 35% of market value) without the voter's approval. The allocation of these revenues among jurisdictions is determined by the relative sizes of their tax revenues in 1929-1933, the last five years in which a prior limit was in effect.

Any tax beyond this amount requires voter approval. To do so, a jurisdiction's governing body must propose the amount of tax revenue it wishes to have the authority to collect and for how long it wants this authority. It can be permanent or expire after a stated number of years. Approval requires a majority of votes. Proposals are typically stated in terms of the tax rate they authorize, but if property values increase, the allowable tax rate is automatically decreased in subsequent years so that the allowable levy continues to produce the same amount of revenue.

When voters approve a new tax levy, it authorizes the jurisdiction to tax up to that level.[1] In practice, jurisdictions almost always use all of this authority for the lifespan of the approved tax. There are many possible explanations for why this is the case in Ohio but not under the other limits we discuss. Some possibilities include that the politicians could prefer far higher taxes than the voters; voters could be perceived to have directly enacted a tax rather than authorized it; or the cost of override referenda could be high.

Using data from the Ohio Department of Taxation on property tax rates and levies we can determine the prevalence and value of overrides of the 1% limit. As stated in the text, 78% of taxing jurisdictions approved new taxes between 2000 and 2016. Voter approved taxes made up 85.9% of the 15.5 billion in property taxes levied in 2014.

It is worth noting that unlike in the other states discussed here, local governments in Ohio can also raise tax

[1] See Ohio Revised Statutes 5705.13-5705.72

revenue with sales and income taxes, and only 65% of local tax revenue comes from the property tax (Urban Institute-Brookings Institution Tax Policy Center 2016). These taxes also require voter approval, though they are not discussed here.

1.3 Wisconsin

Wisconsin restricts the ability of local governments to tax their residents through two separate limits that apply to school districts and to counties and municipalities. The two limits have different structures and voter overrides are more prevalent among school districts.

1.3.1 School districts

Since 1993, Wisconsin has limited the revenues of its school districts to their prior year level of revenue plus a growth rate based on the revenue per student that is occasionally adjusted by the legislature. Typically this adjustment adds to the districts' taxing authority, but in the 2011-12 school year, the legislature instead reduced each district's revenue limit by 5.5%. The limit covers the combined revenues from both the local property tax levy, which is the source of 94.2% of local tax revenue, and state aid. Thus, the limit on a district's local tax levy is the difference between its revenue limit and the amount of state educational aid it will receive. To exceed its limit, a district must specify an amount and duration for the excess. As in Ohio, overrides can be permanent or last a defined number of years. Again, a majority of votes is necessary for approval.

We use data on levy limits from the Wisconsin Department of Public Instruction to determine the frequency and effect of overrides. Each year most districts levy up to their limit. On average, 70% of districts are at their revenue limit and the median school district levied up to their limit in 17 of the 21 years from 1996 to 2016. Voters frequently approve overrides of the limit. As noted in the paper, between 2000 and 2016, 55% of districts passed at least one override. Extended to the earliest year for which data was available, between 1996 and 2016, 59.9% of districts passed at least one override.

Because successful overrides increase the limit for all subsequent years and the growth rate is applied to them in the process, their effects cascade through the years. The combination of permanent and temporary overrides in effect in 2016 resulted in $231 million in higher property taxes, or 5.3% of the total levy limit. Of this, $138 million was due to past and current permanent overrides and the remainder temporary overrides that will eventually expire. Since in any given year, most districts set taxes at their limit this represents approximately the same percentage of actual tax revenues.

It is worth noting that there are some exceptions that allow districts to exceed their limits without voter approval but for limited purposes, for instance if they annex a neighboring district, accept transfer students in an open enrollment program, invest in energy efficiency measures, or lose federal aid. It is certainly possible that the existence of the tax limit on general revenue increases spending in these areas.

1.3.2 Counties and municipalities

Beginning in 2006, state law has limited the tax levy of Wisconsin counties and municipalities to the prior year's tax level plus an allowed increase. The percent increase in the limit is the greater of a growth rate set by the state legislature or the percent increase in the jurisdiction's property values due to new construction. From 2006 through 2010 the growth rate adopted by the legislature was between 2 and 3%, but from 2011 through 2019 it has been 0%. Since the statewide rate of new construction has also diminished, the limit for many governments has declined in real terms.

To exceed their limit, governments must ask their voters to approve an override. As part of a tradition of direct democracy, towns with small populations (under 3000) can vote to approve an override at their annual town meetings. Larger towns must hold a referendum with votes cast at polling places.

Data acquired through a public records request from the Wisconsin Department of Revenue show that between tax years 2009 and 2015, 16% of municipalities exceeded their limits.[2] Interestingly, these were almost exclusively the small towns allowed to approve overrides at their annual town meetings. Only 2 of the 190 cities passed an override. Counties have similarly been infrequent users of overrides; in the same span only 3 of the 72 counties passed at least one override. In total, overrides have occurred 3.45% of the time. From 2016 through 2017, the period for which data is available, counties and municipalities taxed at their limit 72% of the time.[3]

[2] Counties and municipalities may have also passed referenda between 2006 and 2008 however the Wisconsin Department of Revenue could not provide data covering these years. Most jurisdictions had more room within their limits during these years than subsequent years, making it unlikely that overrides are more frequent in years not covered by the data.

[3] There is some unpredictability in their tax levies, this figure includes governments within five dollars of their maximum levy. 78% of governments were within 0.1% of their limit.

2 Discussion of model

The model raises a number of obvious questions which it is useful to briefly discuss. One question is why is the politician biased? In particular, why cannot citizens elect a candidate who shares their tax preferences? Of course, limits would seem unnecessary if elections select the right candidates. Thus, the prevalence of limits at the state and local government level suggests that candidate elections cannot be working perfectly in these environments.[4] To explain this imperfect functioning, it is common to point out that elections at this level of government are small scale affairs and that, as a consequence, citizens are not well informed about candidates' policy preferences. Moreover, the rewards to holding office are not large enough to provide incentives for elected candidates to diverge from their preferences to increase their chances of re-election. But all this only means that, when elected, candidates will likely follow their policy preferences, which may not be congruent with those of the median voter. It does not explain a particular direction of bias. For this, there are (at least) two possible explanations. First, interest groups may put pressure on elected leaders to increase spending above the level preferred by voters. Many stakeholders stand to benefit from public spending. These include public employees, public contractors, and recipients of government grants. By the usual logic of concentrated benefits versus diffuse costs, these stakeholders may form groups to influence politicians. In such environments, politicians may act as if they prefer higher spending even if, as citizens, they share the general voter's preferences (see, for example, Grossman and Helpman 1994 and Besley and Coate 2001). The second explanation is selection. For certain local government offices, it is reasonable to believe that the people most likely to run are those who care intensely about the policies the office controls and thereby have higher preferences for spending on these policies. Good examples might be school board or town and city council.

A related question is how does the designer know the politician's exact bias? Would it not be more realistic to just assume the designer was uncertain of the degree of the politician's bias, allowing in effect a continuous distribution of bias? The answer to this question is obviously yes. A known level of bias is assumed for reasons of tractability.[5] The optimal limit design problem is quite complicated with a known level of politician bias and it makes sense to understand this problem prior to introducing a more general type of uncertainty.

Another question is why is there uncertainty about the citizen's preferred tax? If there were no such

[4] This said, it is possible to generate an explanation for the limits states impose on their local governments while assuming that the median voter theorem applies at both the state and local level. See Calabrese and Epple (2011) and Vigdor (2004).

[5] In the literature on the delegation problem, it is standard to assume a known level of bias. One exception is Amador, Werning, and Angeletos (2006) who provide conditions under which the optimal set of permissible policies remains an interval when bias is uncertain.

uncertainty, the designer should just impose a limit equal to the citizen's preferred tax and that would be the end of the analysis. In particular, there would be no role for overrides. The ubiquity of override provisions suggests that in the real world there must be uncertainty. Moreover, such uncertainty seems intuitively plausible. The type of uncertainty will depend on the nature of the policies the politician is raising taxes to fund. If the policy is road maintenance (snow plowing, pothole repair, etc), then uncertainty would be created by weather, the prices of inputs like road salt, tarmac, etc. If the policy is police protection then uncertainty would be created by the underlying forces generating crime. If the policy is school spending then uncertainty would be created by the prices of school supplies, wages of teachers, mandates from higher levels of government, and state and federal financial support.

A final question is why the citizen does not get his preferred tax once uncertainty is resolved? This reflects the assumption that the politician has agenda-setting power; that is, the politician has the right to choose the tax proposal and citizens only have the right to veto it if it exceeds the limit. The model makes this assumption because it is an accurate description of reality. Alternative arrangements whereby citizens could propose alternatives to the politician's proposal may be possible to design.[6] Intuitively, the underlying reasons why such arrangements may be difficult likely include the problems that i) if citizens could also propose alternatives, it is not clear how to choose between all the alternatives that might be proposed once it is recognized that citizens are heterogeneous in their policy preferences, and ii) the forces that cause elected politicians to be biased might also be expected to result in biased citizen proposals.

3 Proof of assertions made after Proposition 6

In the paper, we make some claims about the solutions to system (16). We note that one solution to system (16) is $H(\ell_r) = 1/4$ and $H(\ell_o) = 3/4$, in which case $(\ell_r + \ell_o)/2 = \tau_m$. We assert this will be the unique solution if the density function $h(\tau)$ is relatively flat (for example, if for all τ' and τ, $h(\tau')/h(\tau) \leq 2$), but without such a condition, there may be multiple solutions and the solution which maximizes the objective function is not necessarily this one. To understand all this, for $x \in (\underline{\tau}, \overline{\tau})$, let $\ell_r(x)$ and $\ell_o(x)$ be defined by

$$H(x) - H(\ell_r) = H(\ell_r) \text{ and } 1 - H(\ell_o) = H(\ell_o) - H(x). \tag{A1}$$

[6]Indeed, the Wisconsin town meeting arrangements discussed in the previous sub-section may permit this.

Note that $\ell_r(x)$ and $\ell_o(x)$ are well defined. The function $\ell_r(x)$ increases from $\underline{\tau}$ to τ_m with derivative

$$\frac{d\ell_r(x)}{dx} = \frac{h(x)}{2h(\ell_r(x))},$$

while the function $\ell_o(x)$ increases from τ_m to $\overline{\tau}$ with derivative

$$\frac{d\ell_o(x)}{dx} = \frac{h(x)}{2h(\ell_o(x))}.$$

Solutions to system (16) correspond to values of x such that

$$x = \frac{\ell_r(x) + \ell_o(x)}{2}. \tag{A2}$$

Observe that the curve $(\ell_r(x) + \ell_o(x))/2$ is greater than x when $x = \underline{\tau}$, is increasing, and is less than x when $x = \overline{\tau}$. It is clear that $x = \tau_m$ satisfies equations (A1) and (A2). It will be the unique solution if the slope of the curve $(\ell_r(x) + \ell_o(x))/2$ is always less than 1. Given the expressions for the derivatives, a sufficient condition for this is that, for all τ' and τ, $h(\tau')/h(\tau) \leq 2$.

If there are multiple solutions, the solution that maximizes the objective function is not necessarily τ_m. To see this, define the function

$$W(x) = \int_{\underline{\tau}}^{\ell_r(x)} [\tau - \ell_r(x)]\, h(\tau)d\tau + \int_{\ell_r(x)}^{x} [\ell_r(x) - \tau]\, h(\tau)d\tau + \int_{x}^{\ell_o(x)} [\tau - \ell_o(x)]\, h(\tau)d\tau + \int_{\ell_o(x)}^{\overline{\tau}} [\ell_o(x) - \tau]\, h(\tau)d\tau.$$

Note that, at an x that corresponds to a solution to system (16), $W(x) = V(\ell_r(x), \ell_o(x))$. Further, observe that

$$\begin{aligned}
W'(x) &= -\int_{\underline{\tau}}^{\ell_r(x)} h(\tau)d\tau \ell_r'(x) + \int_{\ell_r(x)}^{x} h(\tau)d\tau \ell_r'(x) + [\ell_r(x) - x]\, h(x) \\
&\quad - [x - \ell_o(x)]\, h(x) - \int_{x}^{\ell_o(x)} h(\tau)d\tau \ell_o'(x) + \int_{\ell_o(x)}^{\overline{\tau}} h(\tau)d\tau \ell_o'(x). \\
&= \ell_r'(x)\, [H(x) - 2H(\ell_r(x))] + [\ell_r(x) + \ell_o(x) - 2x]\, h(x) + \ell_o'(x)[1 - 2H(\ell_o(x)) + H(x)] \\
&= [\ell_r(x) + \ell_o(x) - 2x]\, h(x).
\end{aligned}$$

Let x^* be some solution different from τ_m. Suppose that $x^* > \tau_m$. Then, if $(\ell(x) + \ell_o(x))/2 > x$ for all $x \in [\tau_m, x^*]$, $W(x^*) > W(\tau_m)$ and $V(\ell(x^*), \ell_o(x^*)) > V(\ell(\tau_m), \ell_o(\tau_m))$. As can be verified diagrammatically, if there are three solutions, one below τ_m and one above, this will be the case and the largest solution will maximize the objective function. With more than three solutions there are additional possibilities, in either case $(\ell(\tau_m), \ell_o(\tau_m))$ is not necessarily the optimal system of limits.

4 Proof of Proposition 7

There are four possibilities for the optimal system of limits (ℓ_r, ℓ_o): i) $\ell_r \leq (1+b)\underline{\tau}$ and $\ell_r \geq (1-b)\ell_o/(1+b)$; ii) $\ell_r > (1+b)\underline{\tau}$ and $\ell_r \geq (1-b)\ell_o/(1+b)$; iii) $\ell_r > (1+b)\underline{\tau}$ and $\ell_r < (1-b)\ell_o/(1+b)$; and iv) $\ell_r \leq (1+b)\underline{\tau}$ and $\ell_r < (1-b)\ell_o/(1+b)$. In case i) the objective function is as described in (15) and, as argued in the text, the optimal system has to satisfy (16). As discussed before Proposition 7, in case ii) the objective function is as described in (17) and the optimal system has to satisfy (19), and in case iii) the objective function is as described in (18) and the optimal system has to satisfy (20). In case iv) the objective function is

$$\begin{array}{c} V(\ell_r, \ell_o) = \int_{\underline{\tau}}^{\ell_r} [\tau - \ell_r]\, h(\tau) d\tau + \int_{\ell_r}^{\frac{\ell_r}{1-b}} [\ell_r - \tau]\, h(\tau) d\tau \\ - \int_{\frac{\ell_r}{1-b}}^{\frac{\ell_o}{1+b}} b\tau h(\tau) d\tau + \int_{\frac{\ell_o}{1+b}}^{\ell_o} [\tau - \ell_o]\, h(\tau) d\tau + \int_{\ell_o}^{\overline{\tau}} [\ell_o - \tau]\, h(\tau) d\tau. \end{array} \tag{A3}$$

The regular limit effectively binds in the range $[\underline{\tau}, \frac{\ell_r}{1-b}]$ and the override limit binds in the range $[\frac{\ell_o}{1+b}, \overline{\tau}]$. The optimal regular limit should therefore be at the median of the truncated distribution on $[\underline{\tau}, \frac{\ell_r}{1-b}]$ and the optimal override limit at the median of the truncated distribution on $[\frac{\ell_o}{1+b}, \overline{\tau}]$. This implies that

$$H(\frac{\ell_r}{1-b}) - H(\ell_r) = H(\ell_r) \text{ and } H(\ell_o) - H(\frac{\ell_o}{1+b}) = 1 - H(\ell_o). \tag{A4}$$

To prove the Proposition, we begin by showing that if the politician's bias b is below $(\overline{\tau} - \underline{\tau})/8\underline{\tau}$, cases i) and iv) are not possible. We start by ruling out case i). As argued in the proof of assertions made after Proposition 6, we know that solutions of system (16) correspond to values of x such that

$$x = \frac{\ell_r(x) + \ell_o(x)}{2},$$

where $(\ell_r(x), \ell_o(x))$ are as defined in (A1). We know that the curve $(\ell_r(x) + \ell_o(x))/2$ is increasing and starts out above the curve x at $\underline{\tau}$ and ends below it at $\overline{\tau}$. In particular, we know that

$$\lim_{x \searrow \underline{\tau}} \frac{\ell_r(x) + \ell_o(x)}{2} = \frac{\underline{\tau} + \tau_m}{2}.$$

Thus, since the curve $\frac{\ell_r(x)+\ell_o(x)}{2}$ is upward sloping, we know that any solution x must exceed $\frac{\underline{\tau}+\tau_m}{2}$. Thus, the smallest value of $\ell(x)$ at a solution is $\ell_r(\frac{\underline{\tau}+\tau_m}{2})$. We claim that $\ell_r(\frac{\underline{\tau}+\tau_m}{2}) \geq \frac{3\underline{\tau}+\tau_m}{4}$. To see this note that $2H(\ell_r(\frac{\underline{\tau}+\tau_m}{2})) = H(\frac{\underline{\tau}+\tau_m}{2})$. Since H is convex on $[\underline{\tau}, \tau_m)$, we have that

$$H(\frac{3\underline{\tau}+\tau_m}{4}) \leq \frac{1}{2}H(\frac{\underline{\tau}+\tau_m}{2}) + \frac{1}{2}H(\underline{\tau}) = \frac{1}{2}H(\frac{\underline{\tau}+\tau_m}{2}).$$

This implies that $\ell_r(\frac{\underline{\tau}+\tau_m}{2}) \geq \frac{3\underline{\tau}+\tau_m}{4}$. This solution requires that $\ell_r(x) \leq (1+b)\,\underline{\tau}$. Given that the smallest possible value of $\ell_r(x)$ is at least $\frac{3\underline{\tau}+\tau_m}{4}$, we cannot have this solution if $(1+b)\underline{\tau} < \frac{3\underline{\tau}+\tau_m}{4}$ which is equivalent to $b < \frac{\tau_m-\underline{\tau}}{4\underline{\tau}} = \frac{\overline{\tau}-\underline{\tau}}{8\underline{\tau}}$.

To rule out case iv), recall that we showed in the proof of Lemma 1 that if $\ell \in (\underline{\tau}, \tau_m)$, it must be the case that

$$H(\frac{\ell}{1-b}) - H(\ell) > H(\ell).$$

This implies that $\ell_r \geq \tau_m$. But then, since $\ell_r < (1-b)\ell_o/(1+b) < \overline{\tau}$

$$2H(\ell_r) \geq 1 > H(\frac{\ell_r}{1-b}).$$

Thus, (A4) has no solution in the relevant range.

We next show that there exists a solution to system (19). As in the proof of assertions made after Proposition 6, for $x \in [\underline{\tau}, \overline{\tau}]$, let $\ell_o(x)$ be defined by

$$1 - H(\ell_o) = H(\ell_o) - H(x).$$

As argued there, $\ell_o(x)$ is well defined and increases smoothly from τ_m to $\overline{\tau}$. Now define the function

$$\varphi(\ell_r) = 2H(\ell_r) - H(\frac{\ell_r}{1+b}).$$

Note that this function is continuous and increasing on $[\underline{\tau}, \tau_m]$ and is such that $\varphi(\underline{\tau}) = 0$ and $\varphi(\overline{\tau}) > 1$. Furthermore, because

$$\varphi'(\ell_r) = 2h(\ell_r) - \frac{h(\frac{\ell_r}{1+b})}{1+b}$$

and h is non-increasing on $[\tau_m, \overline{\tau}]$, $\varphi(\ell_r)$ is not necessarily increasing on $(\tau_m, \overline{\tau}]$. Consider then for all $x \in [\underline{\tau}, \overline{\tau}]$

the equation

$$\varphi(\ell_r) = H(x).$$

Since $\varphi(\underline{\tau}) = 0$, $\varphi(\overline{\tau}) > 1$, and $\varphi(\ell)$ is continuous, this equation has a solution for all x. Because $\varphi(\ell_r)$ is not necessarily increasing on $(\tau_m, \overline{\tau}]$, it may have multiple solutions. Let $\widetilde{\ell}_r(x)$ be defined as the smallest such solution. Note that $\widetilde{\ell}_r(x)$ will be increasing and continuous almost everywhere. However, it may have a discrete number of upward jumps. Finally, because $\varphi(\ell_r)$ is continuous and increasing on $[\underline{\tau}, \tau_m]$, $\widetilde{\ell}_r(x)$ will be continuous on $[\underline{\tau}, x^*)$ where x^* is such that $\varphi(\tau_m) = H(x^*)$.

Define the function on the interval $[\underline{\tau}, \overline{\tau}]$ as follows:

$$f(x) = \frac{\widetilde{\ell}_r(x) + \ell_o(x)}{2}.$$

Solutions to system (19) correspond to values of x such that $f(x) = x$. Note that $f(x)$ is increasing, continuous on $[\underline{\tau}, x^*)$, and continuous almost everywhere. Moreover, $f(\underline{\tau}) > \underline{\tau}$ and $f(\overline{\tau}) < \overline{\tau}$.

We claim that these properties imply that must exist some x such that $f(x) = x$. To see why, let x_1 be the first point of discontinuity of f. Clearly, $x_1 > x^*$. If $f(x) \leq x$ for any $x < x_1$, then, since $f(\underline{\tau}) > \underline{\tau}$, there will exist some $\widetilde{x} \in (\underline{\tau}, x)$ with the required property. Assume therefore that $f(x) > x$ for all $x < x_1$. This means that $\lim_{x \nearrow x_1} f(x) \geq x_1$ which implies that $f(x_1) > x_1$. Now let x_2 be the next point of discontinuity of f. If $f(x) \leq x$ for any $x \in (x_1, x_2)$, then, since $f(x_1) > x_1$, there will exist some $\widetilde{x} \in (x_1, x)$ with the required property. Assume therefore that $f(x) > x$ for all $x \in (x_1, x_2)$. This implies that $\lim_{x \nearrow x_2} f(x) \geq x_2$ which implies that $f(x_2) > x_2$. Continuing on in this way, we must find some points of discontinuity x_{t-1} and x_t for which $f(x_{t-1}) > x_{t-1}$ and $f(x) \leq x$ for some $x \in (x_{t-1}, x_t)$. If not, then $f(x) > x$ for all $x \in [\underline{\tau}, \overline{\tau}]$ which is inconsistent with the fact that $f(\overline{\tau}) < \overline{\tau}$.

It follows from this that system (19) has a solution. Let (ℓ_r^*, ℓ_o^*) be such a solution. It is straightforward to verify that $(1 + b)\,\underline{\tau} < \ell_r^*$. We know that

$$\lim_{x \searrow \underline{\tau}} f(x) = \frac{\underline{\tau} + \tau_m}{2}.$$

Thus, since $f(x)$ is upward sloping, we know that any x such that $f(x) = x$ must have the property that

$x > \frac{\underline{\tau}+\tau_m}{2}$. Thus, the smallest value of $\widetilde{\ell}_r(x)$ at a solution is $\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})$. We know that

$$\varphi(\frac{\underline{\tau}+\tau_m}{2}) = 2H(\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})) - H(\frac{\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})}{1+b}) = H(\frac{\underline{\tau}+\tau_m}{2}).$$

Thus,

$$2H(\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})) > H(\frac{\underline{\tau}+\tau_m}{2})$$

Since H is convex on $[\underline{\tau}, \tau_m)$, we have that

$$H(\frac{3\underline{\tau}+\tau_m}{4}) \leq \frac{1}{2}H(\frac{\underline{\tau}+\tau_m}{2}) + \frac{1}{2}H(\underline{\tau}) = \frac{1}{2}H(\frac{\underline{\tau}+\tau_m}{2}),$$

which implies that $\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) > \frac{3\underline{\tau}+\tau_m}{4}$. The fact that $b < \frac{\tau_m-\underline{\tau}}{4\underline{\tau}} = \frac{\overline{\tau}-\underline{\tau}}{8\underline{\tau}}$ implies that $(1+b)\underline{\tau} < \frac{3\underline{\tau}+\tau_m}{4} < \widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) \leq \ell_r^*$.

Next we show that if at the solution (ℓ_r^*, ℓ_o^*), we have that $\ell_r^* < (1-b)\ell_o^*/(1+b)$ it must be the case that system (20) has a solution in which $\ell_r < (1-b)\ell_o/(1+b)$. Consider first the equation

$$H(\ell_o) - H(\frac{\ell_o}{1+b}) = 1 - H(\ell_o).$$

Note that this corresponds to $\varphi(\ell_o) = H(\overline{\tau})$. We know this equation has solution: namely, $\widetilde{\ell}_r(\overline{\tau})$. Let $\widehat{\ell}_o = \widetilde{\ell}_r(\overline{\tau})$ denote this solution. We claim that $\widehat{\ell}_o > \ell_o^*$. This follows from the fact that

$$1 - H(\ell_o^*) = H(\ell_o^*) - H(\frac{\ell_r^*+\ell_o^*}{2}),$$

and that

$$\frac{\ell_r^*+\ell_o^*}{2} < \frac{\frac{(1-b)\ell_o^*}{1+b}+\ell_o^*}{2} = \frac{\ell_o^*}{1+b},$$

so that

$$H(\frac{\ell_r^*+\ell_o^*}{2}) < H(\frac{\ell_o^*}{1+b}).$$

Next consider the equation

$$H(\ell_r) - H(\frac{\ell_r}{1+b}) = H(\frac{\ell_r}{1-b}) - H(\ell_r).$$

We know that

$$H(\ell_r^*) - H(\frac{\ell_r^*}{1+b}) = H(\frac{\ell_r^*+\ell_o^*}{2}) - H(\ell_r^*).$$

Furthermore, we know that

$$\frac{\ell_r^* + \ell_o^*}{2} > \frac{\ell_r^* + \frac{(1+b)\ell_r^*}{1-b}}{2} = \frac{\ell_r^*}{1-b}.$$

Thus,

$$H(\ell_r^*) - H(\frac{\ell_r^*}{1+b}) > H(\frac{\ell_r^*}{1-b}) - H(\ell_r^*).$$

On the other hand, as demonstrated in the proof of Proposition 3, we have that

$$H(\tau_m) - H(\frac{\tau_m}{1+b}) < H(\frac{\tau_m}{1-b}) - H(\tau_m).$$

Thus, by continuity, there exists a $\widehat{\ell}_r \in (\tau_m, \ell^*)$ such that

$$H(\widehat{\ell}_r) - H(\frac{\widehat{\ell}_r}{1+b}) = H(\frac{\widehat{\ell}_r}{1-b}) - H(\widehat{\ell}_r).$$

Note that

$$\widehat{\ell}_r < \ell_r^* < \frac{(1-b)\ell_o^*}{1+b} < \frac{(1-b)\widehat{\ell}_o}{1+b},$$

as required.

Finally, we note that our solution $(\widehat{\ell}_r, \widehat{\ell}_o)$ is such that $(1+b)\underline{\tau} < \widehat{\ell}_r$. As demonstrated above, $\widehat{\ell}_r > \tau_m$. However, $b < \frac{\tau_m - \underline{\tau}}{4\underline{\tau}}$ implies that

$$\begin{aligned}(1+b)\underline{\tau} &< (1 + \frac{\tau_m - \underline{\tau}}{4\underline{\tau}})\underline{\tau} \\ &= \frac{3\underline{\tau} + \tau_m}{4\underline{\tau}} < \tau_m.\end{aligned}$$

■

5 The relationship between optimal limits

In Section 6, we made the following claim:

Claim *The optimal regular limit is bounded above by the optimal limit with overrides and the optimal override limit is bounded below by the optimal limit without overrides.*

Proof Denote the optimal limit with overrides ℓ, the optimal limit without overrides ℓ_n, and the optimal regular and override limits ℓ_r and ℓ_o. We need to show that $\ell_r \le \ell$ and that $\ell_n \le \ell_o$. There are three possibilities: the optimal system of limits when an override limit is available will either satisfy (16), (19), or (20). We deal with each in turn.

Possibility 1 (ℓ_r, ℓ_o) satisfies (16).

It is immediate that ℓ_r is bounded above by ℓ, since $\ell_r < \tau_m \le \ell$. Thus, we just need to show that ℓ_o is bounded below by ℓ_n. If $\ell_n = \tau_m$, the result holds, since (16) implies that $\ell_o > \tau_m$. If $\ell_n > \tau_m$, then we know from Section 5 that ℓ_n exceeds $(1+b)\underline{\tau}$ and maximizes

$$F(x) = -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x-\tau]\, h(\tau)d\tau.$$

Note that for all $x \in ((1+b)\underline{\tau}, \overline{\tau}]$

$$F'(x) = H\left(\frac{x}{1+b}\right) + 1 - 2H(x).$$

Given that the optimal system of limits satisfies (16), we know that $\ell_r \le (1+b)\underline{\tau}$ and that $\ell_r \ge (1-b)\ell_o/(1+b)$. We also know from (12), (13), and (14), that ℓ_o maximizes

$$G(x) = \begin{cases} \int_{\ell_r}^{\frac{\ell_r+x}{2}} [\ell_r - \tau]\, h(\tau)d\tau + \int_{\frac{\ell_r+x}{2}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x-\tau]\, h(\tau)d\tau \text{ if } x \in [\ell_r, \frac{1+b}{1-b}\ell_r) \\ \int_{\ell_r}^{\frac{\ell_r}{1-b}} [\ell_r - \tau]\, h(\tau)d\tau - \int_{\frac{\ell_r}{1-b}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x-\tau]\, h(\tau)d\tau \text{ if } x \in [\frac{1+b}{1-b}\ell_r, \overline{\tau}] \end{cases}.$$

Note that

$$G'(x) = \begin{cases} H\left(\frac{\ell_r+x}{2}\right) + 1 - 2H(x) \text{ if } x \in [\ell_r, \frac{1+b}{1-b}\ell_r) \\ H\left(\frac{x}{1+b}\right) + 1 - 2H(x) \text{ if } x \in [\frac{1+b}{1-b}\ell_r, \overline{\tau}] \end{cases}.$$

Suppose, to the contrary, that $\ell_n > \ell_o$. Note that if $x \in [\ell_r, \frac{1+b}{1-b}\ell_r)$, we have that

$$H\left(\frac{\ell_r+x}{2}\right) + 1 - 2H(x) \ge H\left(\frac{x}{1+b}\right) + 1 - 2H(x).$$

This implies that $G'(x) \ge F'(x)$ for all $x \ge \ell_r$. Now given that $\ell_o < \ell_n$ and $F(\ell_n) - F(\ell_o) > 0$, we have that

$$0 < F(\ell_n) - F(\ell_o) = \int_{\ell_o}^{\ell_n} F'(x)dx \le \int_{\ell_o}^{\ell_n} G'(x)dx = G(\ell_n) - G(\ell_o).$$

This is a contradiction.

Possibility 2 (ℓ_r, ℓ_o) satisfies (19).

That ℓ_o is bounded below by ℓ_n can be established using a similar argument to that used for Possibility 1. Thus, we just need to show that ℓ_r is bounded above by ℓ. Note first that $\ell_r < \tau_m$, so that if $\ell = \tau_m$, the result is immediate. Thus, we may assume that ℓ exceeds $(1+b)\underline{\tau}$ and maximizes

$$F(x) = \begin{cases} \begin{array}{c} -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\frac{x}{1-b}} [x-\tau]\, h(\tau)d\tau \\ -\int_{\frac{x}{1-b}}^{\overline{\tau}} b\tau h(\tau)d\tau \end{array} & \text{if } x \in [(1+b)\underline{\tau}, (1-b)\,\overline{\tau}) \\ -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x-\tau]\, h(\tau)d\tau & \text{if } x \in ((1-b)\,\overline{\tau}, \overline{\tau}] \end{cases}.$$

Note that

$$F'(x) = \begin{cases} H\left(\frac{x}{1+b}\right) + H\left(\frac{x}{1-b}\right) - 2H(x) & \text{if } x \in [(1+b)\underline{\tau}, (1-b)\,\overline{\tau}) \\ H\left(\frac{x}{1+b}\right) + 1 - 2H(x) & \text{if } x \in ((1-b)\,\overline{\tau}, \overline{\tau}] \end{cases}.$$

Consider the problem of choosing ℓ_r holding constant ℓ_o. We know that $\ell_r > (1+b)\underline{\tau}$ and that $\ell_r \geq (1-b)\ell_o/(1+b)$. We also know from (12), (13), and (14), that ℓ_o maximizes

$$G(x) = \begin{cases} \begin{array}{c} -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\frac{x}{1-b}} [x-\tau]\, h(\tau)d\tau \\ -\int_{\frac{x}{1-b}}^{\frac{\ell_o}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{\ell_o}{1+b}}^{\ell_o} [\tau - \ell_o]\, h(\tau)d\tau \end{array} & \text{if } x \in [(1+b)\underline{\tau}, \frac{1-b}{1+b}\ell_o) \\ \begin{array}{c} -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\frac{x+\ell_o}{2}} [x-\tau]\, h(\tau)d\tau \\ + \int_{\frac{x+\ell_o}{2}}^{\ell_o} [\tau - \ell_o]\, h(\tau)d\tau \end{array} & \text{if } x \in [\frac{1-b}{1+b}\ell_o, \ell_o] \end{cases}.$$

Note that

$$G'(x) = \begin{cases} H(\frac{x}{1-b}) + H(\frac{x}{1+b}) - 2H(x) \text{ if } x \in [(1+b)\underline{\tau}, \frac{1-b}{1+b}\ell_o) \\ H(\frac{x+\ell_o}{2}) + H(\frac{x}{1+b}) - 2H(x) \text{ if } x \in [\frac{1-b}{1+b}\ell_o, \ell_o] \end{cases}.$$

As long as $x \in [\frac{1-b}{1+b}\ell_o, \ell_o]$

$$\begin{aligned} G'(x) &= H(\frac{x+\ell_o}{2}) + H(\frac{x}{1+b}) - 2H(x) \\ &\leq H(\frac{x}{1-b}) + H(\frac{x}{1+b}) - 2H(x). \end{aligned}$$

Suppose to the contrary that $\ell_r > \ell$. Then $G(\ell_r) > G(\ell)$. We know that

$$\begin{aligned} 0 &< G(\ell_r) - G(\ell) = \int_{\ell}^{\ell_r} G'(x)dx \\ &\leq \int_{\ell}^{\ell_r} \left[H(\frac{x}{1-b}) + H(\frac{x}{1+b}) - 2H(x) \right] dx \\ &\leq \int_{\ell}^{\ell_r} F'(x)dx = F(\ell_r) - F(\ell). \end{aligned}$$

This is a contradiction.

Possibility 3: (ℓ_r, ℓ_o) satisfies (20).

We first establish that ℓ_o is bounded below by ℓ_n. Note first that (20) implies that $\ell_o > \tau_m$, so that if $\ell_n = \tau_m$, the result is immediate. We know that $\ell_r > (1+b)\underline{\tau}$ and that $\ell_r \frac{1+b}{1-b} < \ell_o$. From (18), we know that ℓ_o maximizes

$$G(x) = -\int_{\frac{\ell_r}{1-b}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x - \tau]\, h(\tau)d\tau,$$

for all $x \in (\ell_r \frac{1+b}{1-b}, \overline{\tau}]$. If $\ell_n > \tau_m$, then we know from Section 5 that ℓ_n exceeds $(1+b)\underline{\tau}$ and maximizes

$$\begin{aligned} F(x) &= -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\overline{\tau}} [x - \tau]\, h(\tau)d\tau \\ &= G(x) + \int_{\underline{\tau}}^{\frac{\ell_r}{1-b}} b\tau h(\tau)d\tau. \end{aligned}$$

Suppose, to the contrary, that $\ell_o < \ell_n$. Then, it must be the case that $\ell_n \in (\ell_r \frac{1+b}{1-b}, \overline{\tau}]$. This implies that $G(\ell_o) > G(\ell_n)$. But, this also implies that $F(\ell_o) > F(\ell_n)$ - which is a contradiction.

We now show that ℓ_r is bounded above by ℓ. Note first that $\ell_r < \tau_m$, so that if $\ell = \tau_m$, the result is immediate. Thus, given Proposition 1, we can assume that $b < (\overline{\tau} - \underline{\tau})/2\underline{\tau}$. Suppose that $b \in [(\overline{\tau} - \underline{\tau})/2\overline{\tau}, (\overline{\tau} - \underline{\tau})/2\underline{\tau})$ so that Proposition 2 applies. In this case, ℓ is bigger than $(1-b)\overline{\tau}$. This means that it is necessarily bigger than ℓ_r because when the optimal system of limits satisfies (20), we have that $\ell_r < \frac{1-b}{1+b}\ell_o < (1-b)\overline{\tau}$.

Suppose then that $b < (\overline{\tau} - \underline{\tau})/2\overline{\tau}$. By Proposition 3, we know that ℓ is either bigger than $(1-b)\overline{\tau}$ and solves equation (7) or is smaller than $(1-b)\overline{\tau}$ and solves the equation

$$H(\ell) - H\left(\frac{\ell}{1+b}\right) = H\left(\frac{\ell}{1-b}\right) - H(\ell).$$

If it is bigger than $(1-b)\overline{\tau}$, then it is necessarily bigger than ℓ_r because $\ell_r < \frac{1-b}{1+b}\ell_o < (1-b)\overline{\tau}$. If it is smaller than $(1-b)\overline{\tau}$, then we know that ℓ maximizes

$$F(x) = \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\frac{x}{1-b}} [x-\tau]\, h(\tau)d\tau - \int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau - \int_{\frac{x}{1-b}}^{\overline{\tau}} b\tau h(\tau)d\tau.$$

We know that $\ell_r > (1+b)\underline{\tau}$ and that $\ell_r \frac{1+b}{1-b} < \ell_o$. From (18), we know that ℓ_r maximizes

$$\begin{aligned} G(x) &= -\int_{\underline{\tau}}^{\frac{x}{1+b}} b\tau h(\tau)d\tau + \int_{\frac{x}{1+b}}^{x} [\tau - x]\, h(\tau)d\tau + \int_{x}^{\frac{x}{1-b}} [x-\tau]\, h(\tau)d\tau - \int_{\frac{x}{1-b}}^{\frac{\ell_o}{1+b}} b\tau h(\tau)d\tau \\ &\quad + \int_{\frac{\ell_o}{1+b}}^{\ell_o} [\tau - \ell_o]\, h(\tau)d\tau + \int_{\ell_o}^{\overline{\tau}} [\ell_o - \tau]\, h(\tau)d\tau \\ &= F(x) + \int_{\frac{\ell_o}{1+b}}^{\overline{\tau}} b\tau h(\tau)d\tau + \int_{\frac{\ell_o}{1+b}}^{\ell_o} [\tau - \ell_o]\, h(\tau)d\tau + \int_{\ell_o}^{\overline{\tau}} [\ell_o - \tau]\, h(\tau)d\tau, \end{aligned}$$

for all $x \in ((1+b)\underline{\tau}, \frac{1-b}{1+b}\ell_o)$. Suppose, to the contrary, that $\ell_r > \ell$. Then, it must be the case that $\ell \in ((1+b)\underline{\tau}, \frac{1-b}{1+b}\ell_o)$. This implies that $G(\ell_r) > G(\ell)$. But, this also implies that $F(\ell_r) > F(\ell)$ - which is a contradiction. ■

6 Quadratic preferences

In the paper, the politician and citizen have linear distance preferences. In this section, we investigate the robustness of the results in the paper to an alternative specification which is standard in the literature, quadratic distance preferences. If the politician and the citizen have such preferences the main results of the paper are unchanged. While under linear distance preferences, the optimal limit was equal to the median of the probability distribution truncated to the region in which the limit will bind, under quadratic preferences, the optimal limit is instead set at the mean of this truncated distribution. The results presented in the text are otherwise unchanged.

In this specification, when the policy is set at t and the citizen's preferred level is τ, the citizen receives a payoff of $(t-\tau)^2$ and the politician receives $(t-\tau(1+b))^2$. The policy outcomes associated with any given limit remain exactly as described in Section 4.2. What changes is the citizen's expected welfare which is now

$$V(\ell) = -\int_{\underline{\tau}}^{\ell} (\min\{\ell, (1+b)\tau\} - \tau)^2\, h(\tau)d\tau - \int_{\ell}^{\overline{\tau}} (\min\{2\tau - \ell, (1+b)\tau\} - \tau)^2\, h(\tau)d\tau. \qquad \text{(A5)}$$

6.1 Large politician bias

Following the logic of Section 4.3, if the politician's bias exceeds $(\overline{\tau} - \underline{\tau})/\underline{\tau}$ then the voter's welfare for any limit ℓ in the range $[\underline{\tau}, \overline{\tau}]$ is

$$V(\ell) = -\int_{\underline{\tau}}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau.$$

Differentiating this yields

$$V'(\ell) = 2(E[\tau] - \ell).$$

Under the assumptions on h, the optimal limit is equal to τ_m. As in the case of linear preferences, this result can be expanded to a broader range of biases.

Proposition A1. *With quadratic preferences, if the politician's bias exceeds $(\overline{\tau} - \tau_m)/\underline{\tau}$ the optimal limit is τ_m.*

The proof of this result and the ones to follow are provided in Section 8 of the On-line Appendix.

6.2 Small politician bias

With small politician bias, we have the following results which parallel Lemma 1 and Propositions 2 and 3 in the paper.

Lemma A1. *With quadratic preferences, the optimal limit is always at least τ_m.*

Proposition A2. *With quadratic preferences, if the politician's bias is less than $(\overline{\tau} - \tau_m)/\underline{\tau}$ but greater than $(\overline{\tau} - \tau_m)/\overline{\tau}$, the optimal limit solves the equation*

$$\ell = E[\tau | \tau > \ell/(1+b)]. \tag{A6}$$

Proposition A3. *With quadratic preferences, if the politician's bias is less than $(\overline{\tau} - \tau_m)/\overline{\tau}$, the optimal limit is either greater than $\overline{\tau}(1-b)$ and solves equation (A6), or is less than $\overline{\tau}(1-b)$ and solves the equation*

$$\ell = E\left[\tau \middle| \frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right]. \tag{A7}$$

Intuitively, these results can be understood as setting the limit at the optimal policy level given the information about the state of nature conferred by the fact that the citizen is receiving their limit utility. In the case of Proposition A3, the citizen receives their limit utility when τ exceeds $\ell/(1+b)$ but is less than $\ell/(1-b)$.

A marginal increase in the limit from ℓ will benefit the citizen when $\ell < \tau < \ell/(1-b)$ and will hurt when $\ell/(1+b) < \tau < \ell$. Since preferences are quadratic, when the loss and benefit occur, they are not equal but instead increase with the distance between the realization of τ and ℓ. Thus, the optimal limit is set at the mean of the truncated distribution of the citizen's preferred policy level, rather than at the median as was the case with linear preferences.

We also have the following result which parallels Proposition 4.

Proposition A4. *With quadratic preferences, if the politician's bias b is smaller than $(\overline{\tau} - \tau_m)/\overline{\tau}$ and if*

$$\overline{\tau}(1-b) > E\left[\tau \middle| \tau > \frac{\overline{\tau}}{1+2b}\right],$$

the optimal limit is smaller than $(1-b)\overline{\tau}$ and solves equation (A7).

Note that, with no override provision, the optimal limit is τ_m if the politician's bias exceeds $(\overline{\tau} - \tau_m)/\underline{\tau}$, and solves equation (A6) if bias is smaller than $(\overline{\tau} - \tau_m)/\overline{\tau}$. This parallels Proposition 5.

6.3 With an override limit

When an override limit is available, we can obtain results which parallel Propositions 6 and 7. Again, the policy outcomes associated with any given pair of limits remain exactly as described in Section 6. What changes is the citizen's expected welfare which is given by

$$V(\ell_r, \ell_o) = -\int_{\underline{\tau}}^{\ell_r} \left(\min\{\ell_r, (1+b)\tau\} - \tau\right)^2 h(\tau)d\tau - \int_{\ell_r}^{\overline{\tau}} \left(\min\{2\tau - \ell_r, (1+b)\tau, \ell_o\} - \tau\right)^2 h(\tau)d\tau. \quad \text{(A8)}$$

First, we examine what happens the politician's bias is large. If bias is greater than $(\overline{\tau} - \underline{\tau})/\underline{\tau}$ then, by the same logic outlined in Section 6.2 of the paper, the citizen's expected welfare is:

$$V(\ell_r, \ell_o) = -\int_{\underline{\tau}}^{\frac{\ell_r+\ell_o}{2}} (\ell_r - \tau)^2 h(\tau)d\tau - \int_{\frac{\ell_r+\ell_o}{2}}^{\overline{\tau}} (\ell_o - \tau)^2 h(\tau)d\tau. \quad \text{(A9)}$$

Noting that the regular limit binds when τ is less than $\frac{\ell_r+\ell_o}{2}$ and the override limit binds when τ exceeds $\frac{\ell_r+\ell_o}{2}$, we have:

Proposition A5. *With quadratic preferences and an override limit, if the politician's bias exceeds $(\overline{\tau} - \underline{\tau})/\underline{\tau}$,* *the optimal system of limits* (ℓ_r, ℓ_o) *satisfies the equations*

$$\ell_r = E[\tau|\tau < \frac{\ell_r + \ell_o}{2}] \textit{ and } \ell_o = E[\tau|\tau > \frac{\ell_r + \ell_o}{2}]. \tag{A10}$$

Next, we examine what happens when the politician's bias is small. As explained in Section 6.3 of the paper, when bias is below $(\overline{\tau} - \underline{\tau})/8\underline{\tau}$, there are two possible scenarios distinguished by whether $\ell_r/(1-b)$ is larger or smaller than $\ell_o/(1+b)$. In the former case, the citizen's welfare is given by

$$\begin{aligned} V(\ell_r, \ell_o) = \quad & -\int_{\underline{\tau}}^{\frac{\ell_r}{1+b}} (b\tau)^2 h(\tau) d\tau - \int_{\frac{\ell}{1+b}}^{\frac{\ell_r+\ell_o}{2}} (\ell_r - \tau)^2 h(\tau) d\tau \\ & -\int_{\frac{\ell_r+\ell_o}{2}}^{\overline{\tau}} (\ell_o - \tau)^2 h(\tau) d\tau. \end{aligned} \tag{A11}$$

Note that the regular limit effectively binds in the range $[\frac{\ell_r}{1+b}, \frac{\ell_r+\ell_o}{2}]$, while the override limit is binding in the range $[\frac{\ell_r+\ell_o}{2}, \overline{\tau}]$. The optimal system then has the limits at the means of their respective truncated distribution. In the latter case, the citizen's welfare is given by

$$\begin{aligned} V(\ell_r, \ell_o) = \quad & -\int_{\underline{\tau}}^{\frac{\ell_r}{1+b}} (b\tau)^2 h(\tau) d\tau - \int_{\frac{\ell_r}{1+b}}^{\frac{\ell_r}{1-b}} (\ell_r - \tau)^2 h(\tau) d\tau \\ & -\int_{\frac{\ell_r}{1-b}}^{\frac{\ell_o}{1+b}} (b\tau)^2 h(\tau) d\tau - \int_{\frac{\ell_o}{1+b}}^{\overline{\tau}} (\ell_o - \tau)^2 h(\tau) d\tau. \end{aligned} \tag{A12}$$

Note that the regular limit effectively binds in the range $[\frac{\ell_r}{1+b}, \frac{\ell_r}{1-b}]$, while the override limit is binding in the range $[\frac{\ell_o}{1+b}, \overline{\tau}]$. The optimal system then has the limits at the means of their respective truncated distribution. Thus we have:

Proposition A6. *With quadratic preferences and an override limit, if the politician's bias is below* $(\overline{\tau}-\underline{\tau})/8\underline{\tau}$*, the optimal system of limits* (ℓ_r, ℓ_o) *is either such that* $\frac{\ell_r}{1-b} > \frac{\ell_o}{1+b}$ *and satisfies the equations*

$$\ell_r = E[\tau|\frac{\ell_r}{1+b} < \tau < \frac{\ell_r + \ell_o}{2}] \textit{ and } \ell_o = E[\tau|\tau > \frac{\ell_r + \ell_o}{2}], \tag{A13}$$

or *is such that* $\frac{\ell_r}{1-b} < \frac{\ell_o}{1+b}$ *and satisfies the equation*

$$\ell_r = E[\tau|\frac{\ell_r}{1+b} < \tau < \frac{\ell_r}{1-b}] \textit{ and } \ell_o = E[\tau|\tau > \frac{\ell_o}{1+b}]. \tag{A14}$$

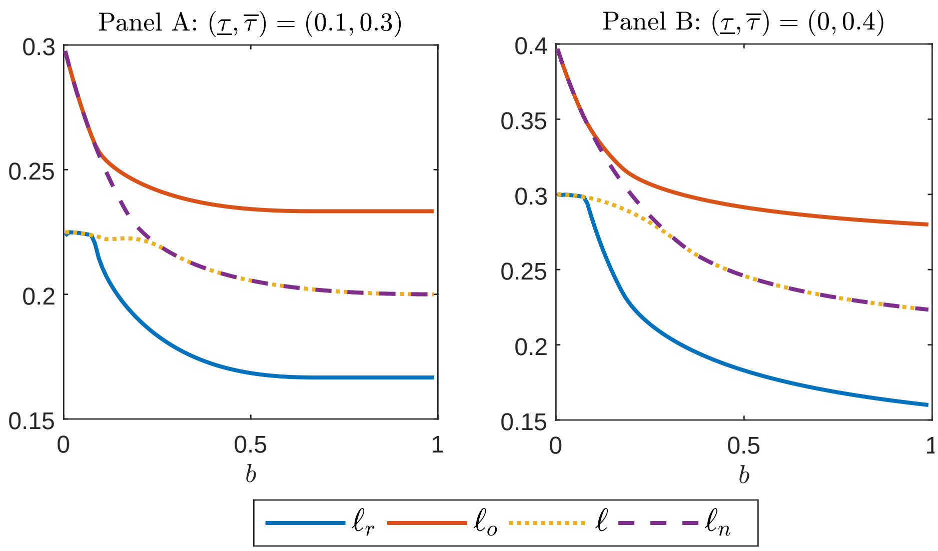

Figure 1: Optimal Limits for the Tent Distribution with Quadratic Preferences

6.4 Example: Tent distribution

To illustrate how the optimal limit depends on the politician's bias with quadratic preferences, Figure 6.4 graphs the optimal limits with overrides, without overrides, and with an override limit for the case where h follows a tent distribution as in Section 7 of the paper. Note that the picture is very similar to that in Figure 2 of the paper. With an override, the optimal limit is not always decreasing in bias. With no override, the optimal limit is decreasing in bias, approaches $\overline{\tau}$ when bias is low and reaches τ_m when bias is high. When an override limit is available, at low levels of bias the optimal system satisfies equation (A14), the optimal regular limit is equal to the optimal limit with overrides, and the override limit is equal to the optimal limit without overrides. At high levels of bias, the optimal system satisfies equation (A10) and $(\ell_r + \ell_o)/2 = \tau_m$. At intermediate levels of bias the optimal system satisfies system (A13) and both limits are decreasing in bias.

7 Constant bias

We next investigate the implications of assuming that the distance between the politician's preferred policy and the citizen's is constant, while returning to linear distance preferences as in the paper. With constant bias, it remains the case that the optimal limit without overrides exceeds the optimal limit with overrides for low bias levels and the case for an override limit remains largely the same. The notable difference from proportional bias is that the optimal limit with overrides is always equal to the median preferred tax level. So while the optimal limit with overrides does not display the *Ally Principle*, it does not decrease as bias decreases either.

In this specification, when the policy is set at t and the citizen's preferred level is τ, the citizen receives a payoff of $-|t-\tau|$ and the politician receives $-|t-(\tau+b)|$. The policy outcomes associated with any given limit now differ from those described in Section 4.2. If $\tau < \ell - b$ the politician will set t at their preferred level, $\tau + b$. If $\ell - b < \tau < \ell$, the citizen would not approve an override at a level greater than ℓ, so the politician will set t at ℓ. If $\ell < \tau < \ell + b$, the politician will propose an override at the highest level the citizen would approve, $2\tau - \ell$. If $\tau > \ell + b$, the politician will propose an override at their preferred level $\tau + b$ and the citizen will approve it. Thus, the citizen's expected welfare will be given by

$$V(\ell) = -\int_{\underline{\tau}}^{\ell} [\min\{\ell, \tau + b\} - \tau]\, h(\tau) d\tau - \int_{\ell}^{\overline{\tau}} [\min\{2\tau - \ell, \tau + b\} - \tau]\, h(\tau) d\tau. \tag{A15}$$

7.1 Large politician bias

If bias were to exceed $\overline{\tau} - \underline{\tau}$ then, even if the citizen's desired tax were at its lowest possible level, the politician would prefer a tax greater than the citizen's highest possible preferred level. In this case, for any limit in the range $[\underline{\tau}, \overline{\tau}]$, $\ell + b$ would exceed $\overline{\tau}$ and $\ell - b$ would be less than $\underline{\tau}$. Hence the citizen's welfare for any limit ℓ would be

$$V(\ell) = \int_{\underline{\tau}}^{\ell} (\tau - \ell) h(\tau) d\tau + \int_{\ell}^{\overline{\tau}} (\ell - \tau) h(\tau) d\tau, \tag{A16}$$

which is exactly the same as that in the large bias case in the paper (i.e., equation (5)). Accordingly, the optimal limit is τ_m. As in the case in the paper, we can weaken the restriction on bias and maintain this conclusion.

Proposition A7. *With constant bias, if $b \geq \tau_m - \underline{\tau}$, the optimal limit is τ_m.*

The logic underlying this result is similar to that for Proposition 1. At the critical level of bias in Proposition A7, it is still the case that a limit at τ_m will produce the same welfare for the citizen as if they ex-ante set the tax at τ_m. This is not the case for alternative possible limits but, nonetheless, the preferred limit remains τ_m.

7.2 Small politician bias

When bias is less than $\tau_m - \underline{\tau}$, it is no longer the case that, if the limit is set at τ_m, the citizen will always get the utility of the limit. For those limits ℓ, like τ_m, where $\ell + b < \overline{\tau}$ and $\ell - b > \underline{\tau}$, the citizen's expected welfare is given by

$$V(\ell) = -\int_{\underline{\tau}}^{\ell-b} bh(\tau)d\tau - \int_{\ell-b}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\ell+b} (\tau - \ell)h(\tau)d\tau - \int_{\ell+b}^{\overline{\tau}} bh(\tau)d\tau.$$

For such limits, the welfare impact of a small change in the limit is

$$V'(\ell) = H(\ell - b) + H(\ell + b) - 2H(\ell). \tag{A17}$$

Hence an optimal limit in the range $[\underline{\tau} + b, \overline{\tau} - b]$ must satisfy

$$H(\ell) - H(\ell - b) = H(\ell + b) - H(\ell). \tag{A18}$$

Since h is symmetric, $\ell = \tau_m$ will always satisfy this condition. However, it is not necessarily the only limit that satisfies condition (A18). For example, if H is uniform, the condition is satisfied by all limits in the interval $[\underline{\tau} + b, \overline{\tau} - b]$.

We can establish the following result about optimal limits other than τ_m.

Lemma A2. *With constant bias, if $\ell \neq \tau_m$ is in the set of optimal limits, $\ell - b$ must be at least $\underline{\tau}$, $\ell + b$ must be no more than $\overline{\tau}$, and h must be constant over the interval $[\ell - b, \ell + b]$.*

Using this result, we can establish that τ_m is always an optimal limit; that is, while there may be other solutions of condition (A18), they cannot generate a higher citizen welfare.

Proposition A8. *With constant bias, τ_m is always in the set of optimal limits.*

This result is in contrast to the case presented in the paper, where the optimal limit changes with bias. Proportional bias makes it more advantageous for the limit to bind when the realization is high than when it is low. If it did not bind, the politician would set the policy at their optimum which is further from the citizen's when the realization is high. With constant bias, this force is removed, and as a result the optimal limit is lower.

7.3 Without an override

Without overrides, if bias exceeds $\overline{\tau} - \underline{\tau}$, the citizen's welfare for any limit ℓ is given by equation (A16) and it follows that the optimal limit is the same as with overrides. In addition, it follows from the proof of Proposition A7 that the optimal limit without overrides is τ_m if bias is at least $\tau_m - \underline{\tau}$. However, if bias is less than $\tau_m - \underline{\tau}$, the optimal limit without overrides will exceed τ_m. Note that if the limit exceeds $\underline{\tau} + b$, the citizen's welfare is

$$V(\ell) = -\int_{\underline{\tau}}^{\ell - b} bh(\tau)d\tau - \int_{\ell - b}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\overline{\tau}} (\tau - \ell)h(\tau)d\tau.$$

Thus, the welfare impact of a small change in the limit is

$$V'(\ell) = 1 + H(\ell - b) - 2H(\ell).$$

Hence, an optimal limit must satisfy

$$H(\ell) - H(\ell - b) = 1 - H(\ell). \tag{A19}$$

Since $V'(\tau_m) > 0$ and $V'(\overline{\tau}) \leq 0$, condition (A19) has a solution on the interval $(\tau_m, \overline{\tau}]$. Moreover, since $V'(\ell)$ is decreasing on $[\underline{\tau}, \tau_m]$ and $V'(\tau_m) > 0$, there is no solution on $[\underline{\tau}, \tau_m]$. As a result, there is always an optimal limit with overrides (τ_m) that is smaller than the optimal limit without. Without overrides, the optimal limit is more stringent the larger the bias.

7.4 With an override limit

Now suppose that an override limit is available and consider the policy outcome that arises under any pair of limits (ℓ_r, ℓ_o) such that $\ell_r \leq \ell_o$. If $\tau < \ell_r - b$ the politician will set t equal to their preferred level, $\tau + b$. If $\ell_r - b < \tau < \ell_r$, the citizen would not approve an override at a level greater than ℓ_r, so the politician will set t equal to ℓ_r. If $\ell_r < \tau$, the politician will propose the minimum of the highest tax the citizen would approve $2\tau - \ell_r$, the override limit ℓ_o, or his preferred level $\tau + b$. Thus, the citizen's expected welfare will be given by

$$V(\ell_r,\ell_o)=-\int_{\underline{\tau}}^{\ell_r}\left[\min\{\ell_r,\tau+b\}-\tau\right]h(\tau)d\tau-\int_{\ell_r}^{\overline{\tau}}\left|\min\{2\tau-\ell_r,\tau+b,\ell_o\}-\tau\right|h(\tau)d\tau. \tag{A20}$$

Two conditions shape the nature of the solution. The first is whether ℓ_r is greater or less than $\underline{\tau}+b$. This determines the policy chosen if τ is less than ℓ_r. The second condition is whether the regular limit ℓ_r is greater or less than ℓ_o-2b, which determines what happens if τ exceeds ℓ_r. If ℓ_r exceeds ℓ_o-2b, then

$$\min\{2\tau-\ell_r,\tau+b,\ell_o\}=\begin{cases}2\tau-\ell_r \text{ if } \tau\in[\ell_r,\frac{\ell_r+\ell_o}{2}]\\ \ell_o \text{ if } \tau\in[\frac{\ell_r+\ell_o}{2},\overline{\tau}]\end{cases}, \tag{A21}$$

while if ℓ_r is less than ℓ_o-2b, then

$$\min\{2\tau-\ell_r,\tau+b,\ell_o\}=\begin{cases}2\tau-\ell_r \text{ if } \tau\in[\ell_r,\ell_r+b]\\ \tau+b \text{ if } \tau\in[\ell_r+b,\ell_o-b]\\ \ell_o \text{ if } \tau\in[\ell_o-b,\overline{\tau}]\end{cases}. \tag{A22}$$

If b is greater than $\overline{\tau}-\underline{\tau}$ then for any pair of limits (ℓ_r,ℓ_o) such that $\ell_r\le\ell_o$ we have that $\ell_r<\underline{\tau}+b$ and $\ell_r>\ell_o-2b$. Thus in this case, we have that the voter's welfare from a set of limits is given by equation (15) in the paper and the optimal system will satisfy the conditions described in Proposition 6.

Proposition A9. *With constant bias and an override limit, if the politician's bias exceeds $\overline{\tau}-\underline{\tau}$, the optimal system of limits (ℓ_r,ℓ_o) satisfies equations (16).*

If the politician's bias is below $(\overline{\tau}-\underline{\tau})/8$, the system of limits described in equations (16) will not be optimal. The logic is similar to that in Section 6.3 of the paper. In this case, equation (16) implies that ℓ_r exceeds $\underline{\tau}+b$ and thus the politician chooses his preferred tax at low realizations of τ. However, the expression for the citizen's welfare in equation (15) assumes that the politician chooses ℓ_r at low realizations of τ. The calculus giving rise to equations (16) is therefore no longer valid. In this situation, there are two possible solutions in both of which ℓ_r exceeds $\underline{\tau}+b$. These two solutions are distinguished by whether ℓ_r is larger or smaller than ℓ_o-2b. In the former case, the citizen's welfare is given by

$$\begin{aligned}V(\ell_r,\ell_o)=&-\textstyle\int_{\underline{\tau}}^{\ell_r-b}bh(\tau)d\tau+\int_{\ell_r-b}^{\ell_r}\left[\tau-\ell_r\right]h(\tau)d\tau+\int_{\ell_r}^{\frac{\ell_r+\ell_o}{2}}\left[\ell_r-\tau\right]h(\tau)d\tau\\&+\textstyle\int_{\frac{\ell_r+\ell_o}{2}}^{\ell_o}\left[\tau-\ell_o\right]h(\tau)d\tau+\int_{\ell_o}^{\overline{\tau}}\left[\ell_o-\tau\right]h(\tau)d\tau.\end{aligned} \tag{A23}$$

Note that the regular limit is effectively binding in the range $[\ell_r - b, \frac{\ell_r+\ell_o}{2}]$, while the override limit is binding in the range $[\frac{\ell_r+\ell_o}{2}, \overline{\tau}]$. The optimal regular limit is therefore at the median of the truncated distribution on $[\ell_r - b, \frac{\ell_r+\ell_o}{2}]$ and the optimal override limit at the median of the truncated distribution on $[\frac{\ell_r+\ell_o}{2}, \overline{\tau}]$. In the latter case, welfare is given by

$$\begin{aligned} V(\ell_r, \ell_o) = &- \textstyle\int_{\underline{\tau}}^{\ell_r-b} bh(\tau)d\tau + \int_{\ell_r-b}^{\ell_r} [\tau - \ell_r]\, h(\tau)d\tau + \int_{\ell_r}^{\ell_r+b} [\ell_r - \tau]\, h(\tau)d\tau \\ &- \textstyle\int_{\ell_r+b}^{\ell_o-b} bh(\tau)d\tau + \int_{\ell_o-b}^{\ell_o} [\tau - \ell_o]\, h(\tau)d\tau + \int_{\ell_o}^{\overline{\tau}} [\ell_o - \tau]\, h(\tau)d\tau. \end{aligned} \tag{A24}$$

The regular limit effectively binds in the range $[\ell_r - b, \ell_r + b]$ and the override limit binds in the range $[\ell_o - b, \overline{\tau}]$. The optimal regular limit is therefore at the median of the truncated distribution on $[\ell_r - b, \ell_r + b]$ and the optimal override limit at the median of the truncated distribution on $[\ell_o - b, \overline{\tau}]$. Thus, we have:

Proposition A10. *With constant bias and an override limit, if the politician's bias b is less than $(\overline{\tau} - \underline{\tau})/8$, the optimal system of limits (ℓ_r, ℓ_o) is* ***either*** *such that $\ell_r \geq \ell_o - 2b$ and satisfies the equations*

$$H(\ell_r) - H(\ell_r - b) = H(\frac{\ell_r + \ell_o}{2}) - H(\ell_r) \text{ and } H(\ell_o) - H(\frac{\ell_r + \ell_o}{2}) = 1 - H(\ell_o), \tag{A25}$$

or *is such that $\ell_r < \ell_o - 2b$ and satisfies the equations*

$$H(\ell_r) - H(\ell_r - b) = H(\ell_r + b) - H(\ell_r) \text{ and } H(\ell_o) - H(\ell_o - b) = 1 - H(\ell_o). \tag{A26}$$

7.5 Example: Tent distribution

To illustrate how the optimal limit depends on the politician's bias with constant bias, Figure 7.5 graphs the optimal limits with overrides, without overrides, and with an override limit for the case where h follows a tent distribution as in Section 7 of the paper. Again, the picture is similar to that in Figure 2 of the paper. The main difference is that, with overrides, the optimal limit is constant and equal to τ_m. With no override, the optimal limit is decreasing in bias, approaches $\overline{\tau}$ when bias is low and reaches τ_m when bias is high. When an override limit is available, at low levels of bias the optimal system satisfies equation (A26), the optimal regular limit is equal to the optimal limit with overrides (τ_m), and the override limit is equal to the optimal limit without overrides. At high levels of bias, the optimal system satisfies equation (16), the optimal regular limit has $H(\ell_r) = .25$, and the optimal override limit has $H(\ell_o) = .75$. At intermediate levels of bias the optimal system satisfies system (A25) and both limits are decreasing in the bias.

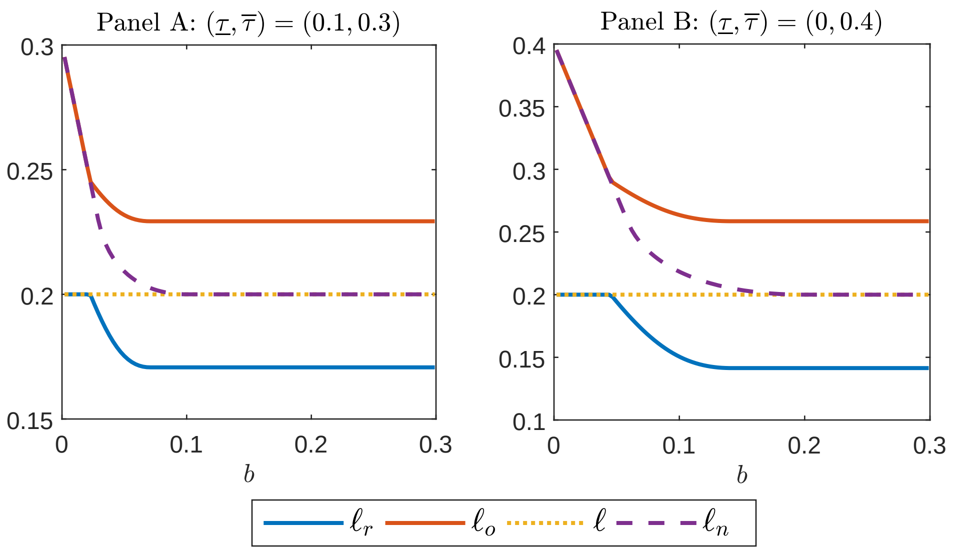

Figure 2: Optimal Limits for the Tent Distribution with Constant Bias

8 Proofs of On-line Appendix Results

8.1 Proof of Proposition A1

We need to show that $V(\tau_m)$ exceeds $V(\ell)$ for any limit in the range $[\underline{\tau}, \tau_m)$ or $(\tau_m, \overline{\tau}]$ where $V(\ell)$ is as defined in equation (A5). For any ℓ in the interval $[\underline{\tau}, \tau_m)$ we know from the condition on the bias that $\ell/(1+b) \leq \underline{\tau}$. There are two possibilities depending on whether $\ell/(1-b)$ exceeds $\overline{\tau}$. If $\ell/(1-b) \geq \overline{\tau}$, then

$$V(\ell) = -\int_{\underline{\tau}}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau$$

and $V(\tau_m)$ must exceed $V(\ell)$ from the argument in Section 6.1 of the On-line Appendix. If instead $\ell/(1-b) < \overline{\tau}$ then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1-b)} (\ell - \tau)^2 h(\tau) d\tau - \int_{\ell/(1-b)}^{\overline{\tau}} (b\tau)^2 h(\tau) d\tau$$

and

$$V(\tau_m) = -\int_{\underline{\tau}}^{\overline{\tau}} (\tau_m - \tau)^2 h(\tau) d\tau.$$

Comparing the two limits, ℓ and τ_m, the citizen would be better off under ℓ than τ_m when $\tau < \frac{\ell+\tau_m}{2}$ and better off under τ_m than ℓ when $\tau > \frac{\ell+\tau_m}{2}$. We can write the difference in the citizen's welfare under the two limits as

$$V(\tau_m) - V(\ell) = \begin{array}{l} -\int_{\underline{\tau}}^{\ell}[(\tau_m-\tau)^2-(\ell-\tau)^2]h(\tau)d\tau - \int_{\ell}^{\tau_m}[(\tau_m-\tau)^2-(\ell-\tau)^2]h(\tau)d\tau \\ -\int_{\tau_m}^{\ell/(1-b)}[(\tau_m-\tau)^2-(\ell-\tau)^2]h(\tau)d\tau - \int_{\ell/(1-b)}^{\overline{\tau}}[(\tau_m-\tau)^2-(b\tau)^2]h(\tau)d\tau. \end{array} \tag{A27}$$

First, we will show that the sum of the first and third terms in equation (A27) is positive. Note first that since

$$b \geq \frac{\overline{\tau}-\tau_m}{\underline{\tau}} > \frac{\tau_m-\underline{\tau}}{\tau_m},$$

even if ℓ was equal to $\underline{\tau}$, $\ell/(1-b)$ would still exceed τ_m. The same condition on bias also means that

$$\frac{\ell}{1-b} > \frac{\ell\tau_m}{\underline{\tau}} - \tau_m + \tau_m > \tau_m + \ell - \underline{\tau}.$$

Consider then the range $[\underline{\tau}, \ell)$ where ℓ is preferable and the range $(\tau_m, \tau_m + \ell - \underline{\tau}]$ where τ_m is preferable. For every τ in the range $[\underline{\tau}, \ell]$, there is a τ' in the range $[\tau_m, \tau_m + \ell - \underline{\tau}]$ for which the benefit to the citizen of the policy being ℓ rather than τ_m when the realization is τ is equal to the benefit to the citizen of the policy being τ_m rather than ℓ when the realization is τ'. This τ' is defined by

$$(\ell-\tau)^2 - (\tau_m-\tau)^2 = (\tau_m-\tau')^2 - (\ell-\tau')^2.$$

Since τ' is as far from τ_m as τ is from ℓ, τ' will always be closer to τ_m than will τ. Thus, since h is non-decreasing on $[\underline{\tau}, \tau_m]$ and symmetric, $h(\tau') \geq h(\tau)$. As a result we have that

$$\int_{\underline{\tau}}^{\ell} \left[(\ell-\tau)^2 - (\tau_m-\tau)^2\right] h(\tau)d\tau - \int_{\tau_m}^{\tau_m+\ell-\underline{\tau}} \left[(\ell-\tau)^2 - (\tau_m-\tau)^2\right] h(\tau)d\tau \geq 0.$$

When the realization of τ is greater than $\tau_m + \ell - \underline{\tau}$ but less than $\ell/(1-b)$, the citizen is strictly better off with the policy at τ_m rather than ℓ so the inequality above holds strictly if we expand the second integral to include this range. Thus we have,

$$\int_{\underline{\tau}}^{\ell} \left[(\ell-\tau)^2 - (\tau_m-\tau)^2\right] h(\tau)d\tau - \int_{\tau_m}^{\ell/(1-b)} \left[(\ell-\tau)^2 - (\tau_m-\tau)^2\right] h(\tau)d\tau > 0.$$

Next, we contend that the second term in equation (A27) is non-negative. Over the range $[\ell, \tau_m]$, the citizen has a higher payoff under ℓ when τ is less than $(\ell+\tau_m)/2$ and a higher payoff under τ_m when τ is greater than $(\ell + \tau_m)/2$. Thus,

$$\int_{\ell}^{\tau_m} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2 \right] d\tau = 0$$

However because h is non-decreasing over the range $[\ell, \tau_m]$,

$$\int_{\ell}^{\tau_m} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2 \right] h(\tau) d\tau \geq 0.$$

It remains to show that the fourth term in equation (A27) is also positive. To see that this is true, note that the condition on the bias implies that $(b\tau)^2 \geq \left(\tau \left(\frac{\overline{\tau} - \tau_m}{\overline{\tau}}\right)\right)^2$. Since $(b\tau)^2$ is increasing in τ, for any τ in the interval $[\ell/(1-b), \overline{\tau}]$, $(b\tau)^2$ is thus at least $(\overline{\tau} - \tau_m)^2$, while $(\tau_m - \tau)^2$ cannot exceed $(\tau_m - \overline{\tau})^2$. Thus $(b\tau)^2 - (\tau_m - \tau)^2$ is at least zero for all τ in the range. As a result, we have that $V(\tau_m) - V(\ell)$ strictly exceeds zero.

For ℓ in the interval $(\tau_m, \overline{\tau}]$ if $\ell/(1+b) \leq \underline{\tau}$ then

$$V(\ell) = -\int_{\underline{\tau}}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau$$

and $V(\tau_m)$ must exceed $V(\ell)$ from the argument in Section 6.1 of the On-line Appendix. If instead $\ell/(1+b) > \underline{\tau}$ then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau) d\tau - \int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau,$$

and we can write the difference between the citizen's expected welfare under the two limits as

$$\begin{aligned} V(\tau_m) - V(\ell) = \; & \textstyle\int_{\underline{\tau}}^{\ell/(1+b)} [(b\tau)^2 - (\tau_m - \tau)^2] h(\tau) d\tau - \int_{\ell/(1+b)}^{\tau_m} [(\tau_m - \tau)^2 - (\ell - \tau)^2] h(\tau) d\tau \\ & - \textstyle\int_{\tau_m}^{\ell} [(\tau_m - \tau)^2 - (\ell - \tau)^2] h(\tau) d\tau - \int_{\ell}^{\overline{\tau}} [(\tau_m - \tau)^2 - (\ell - \tau)^2] h(\tau) d\tau. \end{aligned} \tag{A28}$$

First, we will show that the sum of the second and fourth terms in equation (A28) is positive. Note first that since

$$b \geq \frac{\overline{\tau} - \tau_m}{\underline{\tau}} > \frac{\overline{\tau} - \tau_m}{\ell - \tau_m + \underline{\tau}},$$

we have that

$$\frac{\ell}{1+b} < \frac{\ell - \tau_m + \underline{\tau}}{\overline{\tau} + \underline{\tau} - 2\tau_m + \ell} \ell.$$

Since $\tau_m - \underline{\tau} = \overline{\tau} - \tau_m$, the right hand side of the inequality above is equal to $\ell - \tau_m + \underline{\tau}$. Thus,

$$\tau_m - \frac{\ell}{1+b} > \tau_m - (\ell - \tau_m + \underline{\tau}) = \overline{\tau} - \ell.$$

For every τ in the range $[\ell, \overline{\tau}]$, define $\tau' \in [\tau_m - (\overline{\tau} - \ell), \tau_m]$ with

$$(\ell - \tau)^2 - (\tau_m - \tau)^2 = (\tau_m - \tau')^2 - (\ell - \tau')^2. \tag{A29}$$

The benefit to the citizen of the policy being ℓ rather than τ_m when the realization is τ is equal to the benefit to the citizen of the policy being τ_m rather than ℓ when the realization is τ'. Since τ' is as far from τ_m as τ is from ℓ, τ' will always be closer to τ_m than will τ. Thus, since h is non-decreasing on $[\underline{\tau}, \tau_m]$ and symmetric, $h(\tau') \geq h(\tau)$. As a result we have that

$$\int_{\tau_m - (\overline{\tau} - \ell)}^{\tau_m} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] h(\tau) d\tau - \int_{\ell}^{\overline{\tau}} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] h(\tau) d\tau \geq 0.$$

When the realization of τ is between $\ell/(1+b)$ and $\tau_m - (\overline{\tau} - \ell)$, the citizen is strictly better off with the policy at τ_m rather than ℓ so the inequality above holds strictly if we expand the first integral to include this range. Thus we have,

$$\int_{\ell/(1+b)}^{\tau_m} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] h(\tau) d\tau - \int_{\ell}^{\overline{\tau}} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] h(\tau) d\tau > 0.$$

Next, we contend that the third term in equation (A28) is non-negative. Over the range $[\tau_m, \ell]$, the citizen has a higher payoff under ℓ when τ is greater than $(\ell + \tau_m)/2$ and a higher payoff under τ_m when τ is less than $(\ell + \tau_m)/2$. Thus,

$$\int_{\tau_m}^{\ell} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] d\tau = 0.$$

However, because h is non-increasing over the range $[\tau_m, \ell]$,

$$\int_{\tau_m}^{\ell} \left[(\ell - \tau)^2 - (\tau_m - \tau)^2\right] h(\tau) d\tau \geq 0.$$

It remains to show that the first term in equation (A28) is also positive. To see that this is true, note that the condition on the bias implies that $(b\tau)^2 \geq \left(\tau \left(\frac{\overline{\tau} - \tau_m}{\underline{\tau}}\right)\right)^2$. Since $(b\tau)^2$ is increasing in τ, for any τ in the interval $[\underline{\tau}, \ell/(1+b))$, $(b\tau)^2$ is thus at least $(\overline{\tau} - \tau_m)^2$, while $(\tau_m - \tau)^2$ cannot exceed $(\tau_m - \underline{\tau})^2$. Thus

$(b\tau)^2 - (\tau_m - \tau)^2$ is at least zero for all τ in the range. As a result, we have that $V(\tau_m) - V(\ell)$ strictly exceeds zero. ■

8.2 Proof of Lemma A1

Proposition A1 establishes this result for $b \geq (\overline{\tau} - \tau_m)/\underline{\tau}$ so it remains to show this for lesser levels of bias. Consider some limit $\ell < \tau_m$ we will show that marginally increasing ℓ will increase the citizen's payoff.

Suppose first that $\ell \geq (1-b)\overline{\tau}$. If $\ell \geq (1+b)\underline{\tau}$, then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau) d\tau - \int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau.$$

Note that

$$V'(\ell) = -2\int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau) h(\tau) d\tau = 2\left(1 - H\left(\frac{\ell}{1+b}\right)\right)(E[\tau|\tau > \ell/1+b] - \ell) > 0$$

which implies that raising the limit slightly will increase the citizen's payoff. If $\ell < (1+b)\underline{\tau}$ then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\overline{\tau}} (\ell - \tau)^2 h(\tau) d\tau$$

and from the argument in Section 6.1 of the On-line Appendix we know that $V(\tau_m)$ exceeds this.

Now suppose that $\ell < (1-b)\overline{\tau}$. If $\ell \geq (1+b)\underline{\tau}$, then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau) d\tau - \int_{\ell/(1+b)}^{\ell/(1-b)} (\ell - \tau)^2 h(\tau) d\tau - \int_{\ell/(1-b)}^{\overline{\tau}} (b\tau)^2 h(\tau) d\tau.$$

Note that

$$V'(\ell) = -2\int_{\ell/(1+b)}^{\ell/(1-b)} (\ell - \tau) h(\tau) d\tau = 2\left(H\left(\frac{\ell}{1-b}\right) - H\left(\frac{\ell}{1+b}\right)\right)\left(E\left[\tau \middle| \frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right] - \ell\right).$$

First, suppose that $\ell/(1-b) \leq \tau_m$. In this case, since h is non-decreasing on the interval $[\underline{\tau}, \tau_m)$, it must be the case that

$$E\left[\tau | \frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right] \geq \frac{1}{2}\left(\frac{\ell}{1+b} + \frac{\ell}{1-b}\right) = \frac{\ell}{1-b^2} > \ell.$$

As a result, marginally increasing the limit would benefit the citizen. If instead $\ell/(1-b) > \tau_m$, consider first the case where $\ell/(1+b)$ is at least as close to τ_m than is $\ell/(1-b)$ so, $\tau_m - \ell/(1+b) \leq \ell/(1-b) - \tau_m$. Since h is symmetric we have that

$$E[\tau|\frac{\ell}{1+b} < \tau < 2\tau_m - \frac{\ell}{1+b}] = \tau_m > \ell.$$

Since

$$\frac{\ell}{1-b} > 2\tau_m - \frac{\ell}{1+b},$$

we have that

$$E[\tau|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}] > \tau_m > \ell,$$

and marginally increasing the limit would improve the citizen's expected welfare. Next we show that it is not possible for ℓ to be optimal and have $\ell/(1-b)$ closer to τ_m than is $\ell/(1+b)$. Suppose instead that it were, so $\ell/(1-b) - \tau_m < \tau_m - \ell/(1+b)$ and

$$E[\tau|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}] \leq \ell.$$

Since

$$2\ell - \frac{\ell}{1+b} < \frac{\ell}{1-b},$$

we have that

$$E[\tau|\frac{\ell}{1+b} < \tau < 2\ell - \frac{\ell}{1+b}] < E[\tau|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}].$$

Hence,

$$E[\tau|\frac{\ell}{1+b} < \tau < 2\ell - \frac{\ell}{1+b}] < \ell.$$

Since the interval $[\ell/(1+b), 2\ell - \ell/(1+b)]$ is centered around ℓ this is not possible given that $\ell < \tau_m$. Thus it must be the case that

$$E\left[\tau|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right] > \ell,$$

and a small increase in the limit would benefit the citizen. If $\ell < (1+b)\underline{\tau}$ then we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1-b)} (\ell-\tau)^2 h(\tau) d\tau - \int_{\ell/(1-b)}^{\overline{\tau}} (b\tau)^2 h(\tau) d\tau.$$

Differentiating shows that the effect of a marginal change in the limit on the citizen's expected welfare is

$$V'(\ell) = -2\int_{\underline{\tau}}^{\ell/(1-b)} (\ell - \tau)h(\tau)d\tau = 2H\left(\frac{\ell}{1-b}\right)\left(E\left[\tau|\tau < \frac{\ell}{1-b}\right] - \ell\right).$$

First, suppose that $\ell/(1-b) < \tau_m$. In this case, since h is non-decreasing on the interval $[\underline{\tau}, \tau_m)$, it must be the case that

$$E\left[\tau|\tau < \frac{\ell}{1-b}\right] \geq \frac{1}{2}\left(\underline{\tau} + \frac{\ell}{1-b}\right).$$

Since

$$\frac{1}{2}\left(\frac{\ell}{1+b} + \frac{\ell}{1-b}\right) > \ell,$$

and

$$\frac{\ell}{1+b} < \underline{\tau},$$

we have that

$$E\left[\tau|\tau < \frac{\ell}{1-b}\right] > \ell.$$

So, marginally increasing the limit would benefit the citizen. If instead $\ell/(1-b) \geq \tau_m$, consider the interval $[\underline{\tau}, 2\ell - \underline{\tau}]$, which is centered around ℓ. Since h is non-decreasing on the interval $[\underline{\tau}, \tau_m]$ and symmetric, it must be the case that

$$E[\tau|\tau < 2\ell - \underline{\tau}] \geq \ell.$$

The hypothesis that $\ell/(1+b) < \underline{\tau}$ implies that $\ell/(1-b)$ exceeds $2\ell - \underline{\tau}$. Thus,

$$E\left[\tau|\tau < \frac{\ell}{1-b}\right] > E[\tau|\tau < 2\ell - \underline{\tau}],$$

and hence

$$E\left[\tau|\tau < \frac{\ell}{1-b}\right] > \ell,$$

which again implies that marginally increasing the limit would benefit the citizen. ■

8.3 Proof of Proposition A2

For limits $\ell \in [\tau_m, \overline{\tau}]$, we have that $\ell/(1+b)$ is greater than or equal to $\tau_m/(1+b)$ which since b is less than $(\tau_m - \underline{\tau})/\underline{\tau}$ exceeds $\underline{\tau}$. In addition, if $b < 1$, we have that $\ell/(1-b)$ is greater than or equal to $\tau_m/(1-b)$

which, since b exceeds $(\overline{\tau} - \tau_m)/\overline{\tau}$, exceeds $\overline{\tau}$. Thus for limits $\ell \in [\tau_m, \overline{\tau}]$ we have that

$$V(\ell) = -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau)d\tau - \int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau)^2 h(\tau)d\tau.$$

This means that

$$V'(\ell) = -2\int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau)h(\tau)d\tau = 2\left(1 - H\left(\frac{\ell}{1+b}\right)\right)\left(E\left[\tau \middle| \tau > \frac{\ell}{1+b}\right] - \ell\right).$$

It follows that at the optimal limit

$$\ell = E\left[\tau \middle| \tau > \frac{\ell}{1+b}\right].$$

To see that this equation has a solution note that

$$\tau_m < E\left[\tau \middle| \tau > \frac{\tau_m}{1+b}\right],$$

and that

$$\overline{\tau} > E\left[\tau \middle| \tau > \frac{\overline{\tau}}{1+b}\right].$$

Thus, by the Intermediate Value Theorem there exists a solution to equation (A6). ■

8.4 Proof of Proposition A3

For limits $\ell \in [\tau_m, \overline{\tau}]$, we have that $\ell/(1+b)$ is greater than or equal to $\tau_m/(1+b)$ which, since b is less than $(\overline{\tau} - \tau_m)/\overline{\tau}$ exceeds $\underline{\tau}$. Moreover, since $\tau_m/(1-b)$ is less than $\overline{\tau}$ which is less than $\overline{\tau}/(1-b)$, we have that

$$\frac{\ell}{1-b} \gtreqless \overline{\tau} \text{ as } \ell \gtreqless (1-b)\overline{\tau}.$$

It follows then that the citizen's welfare with limit $\ell \in [\tau_m, \overline{\tau}]$ is

$$V(\ell) = \begin{cases} -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau)d\tau - \int_{\ell/(1+b)}^{\overline{\tau}} (\ell - \tau)^2 h(\tau)d\tau \text{ if } \ell < (1-b)\overline{\tau} \\ -\int_{\underline{\tau}}^{\ell/(1+b)} (b\tau)^2 h(\tau)d\tau - \int_{\ell/(1+b)}^{\ell/(1-b)} (\ell - \tau)^2 h(\tau)d\tau - \int_{\ell/(1-b)}^{\overline{\tau}} (b\tau)^2 h(\tau)d\tau \text{ if } \ell \geq (1-b)\overline{\tau}. \end{cases}$$

Thus the impact on welfare of a small increase in the limit is

$$V'(\ell) = \begin{cases} 2\left(E\left[\tau|\tau > \frac{\ell}{1+b}\right] - \ell\right) \text{ if } \ell < (1-b)\overline{\tau} \\ 2\left(E\left[\tau\Big|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right] - \ell\right) \text{ if } \ell \geq (1-b)\overline{\tau}. \end{cases}$$

It follows that the optimal limit is either such that $\ell \in [\tau_m, (1-b)\overline{\tau})$ and solves

$$\ell = E\left[\tau|\tau > \frac{\ell}{1+b}\right],$$

or is such that $\ell \in [(1-b)\overline{\tau}, \overline{\tau}]$ and solves

$$\ell = E\left[\tau\Big|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right].$$

It is straightforward to show that at least one of these equations must have a solution in the relevant range. From the Proof of Proposition A1 we know that equation (A6) must have a solution on the range $(\tau_m, \overline{\tau}]$. Suppose for all such solutions it is the case that $\ell < (1-b)\overline{\tau}$, then it must be the case that

$$(1-b)\overline{\tau} > E\left[\tau\Big|\tau > \frac{\ell}{1+b}\right] \Rightarrow (1-b)\overline{\tau} > E\left[\tau\Big|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right].$$

Since h is symmetric around τ_m and the interval $[\tau_m/(1+b), 2\tau_m - \tau_m/(1+b))$ has a midpoint equal to τ_m, it must be the case that

$$\tau_m = E\left[\tau\Big|\frac{\tau_m}{1+b} < \tau < \tau_m + \left(\tau_m - \frac{\tau_m}{1+b}\right)\right].$$

Since $\tau_m/(1-b) > \tau_m + \left(\tau_m - \frac{\tau_m}{1+b}\right)$ it must be the case that

$$E\left[\tau\Big|\frac{\tau_m}{1+b} < \tau < \tau_m + \left(\tau_m - \frac{\tau_m}{1+b}\right)\right] \leq E\left[\tau\Big|\frac{\tau_m}{1+b} < \tau < \frac{\tau_m}{1-b}\right]$$

and hence

$$\tau_m \leq E\left[\tau\Big|\frac{\tau_m}{1+b} < \tau < \frac{\tau_m}{1-b}\right].$$

Thus, since $E[\tau|\frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}]$ is continuous, greater than ℓ when ℓ is equal to τ_m, and less than ℓ when ℓ

is equal to $(1-b)\overline{\tau}$, by the Intermediate Value Theorem, there must exist an $\ell \in [\tau_m, (1-b)\overline{\tau}]$ such that

$$\ell = E\left[\tau \middle| \frac{\ell}{1+b} < \tau < \frac{\ell}{1-b}\right].$$

■

8.5 Proof of Proposition A4

Given Proposition A3, it suffices to show that, under the condition of the Proposition, equation (A6) has no solution bigger than $(1-b)\overline{\tau}$. Suppose then that ℓ solves equation (A6). We first observe that ℓ must be less than or equal to $(1+b)\overline{\tau}/(1+2b)$. To see this note that since h is non-increasing on $(\tau_m, \overline{\tau}]$

$$E\left[\tau \middle| \tau > \frac{\ell}{1+b}\right] \leq \frac{1}{2}\left(\overline{\tau} + \frac{\ell}{1+b}\right).$$

Since the left hand side of this inequality must equal ℓ, we have that

$$\ell \leq \overline{\tau}\frac{1+b}{1+2b}.$$

We now show that ℓ must be less than $(1-b)\overline{\tau}$. Suppose not. Then it must be the case that $\ell \in [(1-b)\overline{\tau}, (1+b)\overline{\tau}/(1+2b)]$. We then have that

$$E\left[\tau|\tau > \frac{\ell}{1+b}\right] < E\left[\tau|\tau > \frac{\overline{\tau}}{1+2b}\right],$$

which by the condition of the Proposition tells us that

$$E\left[\tau|\tau > \frac{\ell}{1+b}\right] < \overline{\tau}(1-b).$$

Since $\ell > \overline{\tau}(1-b)$, this contradicts the fact that ℓ solves equation (A6). ■

8.6 Proof of Proposition A6

There are four possibilities for the optimal system of limits (ℓ_r, ℓ_o): i) $\ell_r \leq (1+b)\,\underline{\tau}$ and $\ell_r \geq (1-b)\ell_o/(1+b)$; ii) $\ell_r > (1+b)\,\underline{\tau}$ and $\ell_r \geq (1-b)\ell_o/(1+b)$; iii) $\ell_r > (1+b)\,\underline{\tau}$ and $\ell_r < (1-b)\ell_o/(1+b)$; and iv) $\ell_r \leq (1+b)\,\underline{\tau}$

and $\ell_r < (1-b)\ell_o/(1+b)$. In case i) the objective function is as described in (A9) and, as argued in the text, the optimal system has to satisfy (A10). As discussed before Proposition A6, in case ii) the objective function is as described in (A11) and the optimal system has to satisfy (A13), and in case iii) the objective function is as described in (A12) and the optimal system has to satisfy (A14). In case iv) the objective function is

$$V(\ell_r, \ell_o) = \begin{array}{l} -\int_{\underline{\tau}}^{\frac{\ell_r}{1-b}} (\ell_r - \tau)^2 h(\tau) d\tau - \int_{\frac{\ell_r}{1-b}}^{\frac{\ell_o}{1+b}} (b\tau)^2 h(\tau) d\tau \\ -\int_{\frac{\ell_o}{1+b}}^{\overline{\tau}} (\tau - \ell_o)^2 h(\tau) d\tau. \end{array} \tag{A30}$$

This implies that

$$\ell_r = E\left[\tau | \tau < \frac{\ell_r}{1-b}\right] \text{ and } \ell_o = E\left[\tau | \tau > \frac{\ell_o}{1+b}\right]. \tag{A31}$$

We begin by showing that if the politician's bias is below $(\overline{\tau} - \underline{\tau})/8\tau$, cases i) and iv) are not possible. We start by ruling out case i). Define $\ell_r(x)$ and $\ell_o(x)$ as

$$\ell_r(x) = E[\tau | \tau < x] \text{ and } \ell_o(x) = E[\tau | \tau > x].$$

The function $\ell_r(x)$ increases from $\underline{\tau}$ to τ_m with derivative

$$\frac{d\ell_r(x)}{dx} = \frac{h(x)}{H(x)}(x - \ell_r(x)) \geq 0.$$

Then solutions to system (A10) correspond to values of x such that

$$x = \frac{\ell_r(x) + \ell_o(x)}{2}.$$

The function $(\ell_r(x) + \ell_o(x))/2$ is increasing, greater than x when $x = \underline{\tau}$, and less than x when $x = \overline{\tau}$. In particular, we know that

$$\lim_{x \searrow \underline{\tau}} \frac{\ell_r(x) + \ell_o(x)}{2} = \frac{\underline{\tau} + \tau_m}{2}.$$

Thus, any solution x must exceed $\frac{\underline{\tau}+\tau_m}{2}$ and $\ell_r(x)$ at a solution must be at least $\ell_r(\frac{\underline{\tau}+\tau_m}{2})$. Since h is non-decreasing on the interval $[\underline{\tau}, \tau_m]$ the lowest value that $E[\tau | \tau < \frac{\underline{\tau}+\tau_m}{2}]$ can take on is if h is uniform in which case it will be equal to $\frac{3\underline{\tau}+\tau_m}{4}$. Hence ℓ_r will be at least this level. Thus we cannot have this solution if $\underline{\tau}(1+b) < \frac{3\underline{\tau}+\tau_m}{4}$, which is equivalent to $b < \frac{\overline{\tau}-\underline{\tau}}{8\underline{\tau}}$

To rule out case iv) recall that we showed in Lemma A1 that for $\ell < \tau_m$, $E[\tau|\tau < \frac{\ell}{1-b}] > \ell$. This implies that $\ell_r \geq \tau_m$. But since $\frac{\ell_r}{1-b} < \frac{\ell_o}{1+b} < \overline{\tau}$ which implies that

$$E[\tau|\tau < \frac{\ell_r}{1-b}] < E[\tau|\tau < \frac{\ell_o}{1-b}] < E[\tau|\tau < \overline{\tau}] = \tau_m.$$

Thus system (A31) has no solution in the relevant range.

We next show that there exists a solution to system (A13). For $x \in [\underline{\tau}, \overline{\tau}]$, let $\ell_o(x) = E[\tau|\tau > x]$. As argued above, $\ell_o(x)$ is well defined and increases smoothly from τ_m to $\overline{\tau}$. Now define the function

$$\varphi(x, \ell_r) = E[\tau|\frac{\ell_r}{1+b} < \tau < x].$$

Note that because h is continuous, for any x, this function is continuous in ℓ_r. Furthermore it is increasing in ℓ_r on $[\underline{\tau}, x)$, has $\varphi(x, \underline{\tau}) = E[\tau|\tau < x] \geq \underline{\tau}$, and has $\varphi(x, x) = E[\tau|\frac{x}{1+b} < \tau < x] < x$. Thus by the intermediate value theorem $\varphi(x, \ell_r) = \ell_r$ must have a solution where $\ell_r \in [\underline{\tau}, x)$. Because $\partial\varphi(x, \ell_r)/\partial\ell_r$ is not necessarily greater than one over the relevant range, there may be multiple solutions. Let $\widetilde{\ell}_r(x)$ be defined as the smallest such solution. Note that $\widetilde{\ell}_r(x)$ will be increasing and continuous almost everywhere. However, it may have a discrete number of upward jumps.

Define the function on the interval $[\underline{\tau}, \overline{\tau}]$ as follows:

$$f(x) = \frac{\widetilde{\ell}_r(x) + \ell_o(x)}{2}.$$

Solutions to system (A13) correspond to values of x such that $f(x) = x$. Note that $f(x)$ is increasing, continuous almost everywhere, has $f(\underline{\tau}) > \underline{\tau}$ and $f(\overline{\tau}) < \overline{\tau}$. By the argument in the Proof of Proposition 7, this implies there must exist some x such that $f(x) = x$. It follows that system (A13) has a solution. Let (ℓ_r^*, ℓ_o^*) be such a solution. It is straightforward to verify that $(1+b)\underline{\tau} < \ell_r^*$. We know that

$$\lim_{x \searrow \underline{\tau}} f(x) = \frac{\underline{\tau} + \tau_m}{2}.$$

Thus, since $f(x)$ is upward sloping, we know that any x such that $f(x) = x$ must have the property that

$x > \frac{\underline{\tau}+\tau_m}{2}$. Thus, the smallest value of $\ell(x)$ at a solution is $\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})$. We know that

$$\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) = E\left[\tau|\frac{\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2})}{1+b} < \tau < \frac{\underline{\tau}+\tau_m}{2}\right] \geq E\left[\tau|\tau < \frac{\underline{\tau}+\tau_m}{2}\right] \geq \frac{\underline{\tau}+\frac{\underline{\tau}+\tau_m}{2}}{2}.$$

Since h is non-decreasing on $[\underline{\tau}, \tau_m]$, it must be the case that

$$E\left[\tau|\tau < \frac{\underline{\tau}+\tau_m}{2}\right] \geq \frac{\underline{\tau}+\frac{\underline{\tau}+\tau_m}{2}}{2} = \frac{3\underline{\tau}+\tau_m}{4}.$$

Which implies that $\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) \geq \frac{3\underline{\tau}+\tau_m}{4}$. The fact that $b < \frac{\tau_m-\underline{\tau}}{4\underline{\tau}} = \frac{\overline{\tau}-\underline{\tau}}{8\underline{\tau}}$ implies that $(1+b)\underline{\tau} < \frac{3\underline{\tau}+\tau_m}{4} \leq \widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) \leq \ell_r^*$.

Next we show that if at the solution (ℓ_r^*, ℓ_o^*), we have that $\ell_r^*/(1-b) < \ell_o^*/(1+b)$ it must be the case that system (A14) has a solution in which $\ell_r/(1-b) < \ell_o/(1+b)$. Consider first the expression

$$E\left[\tau|\tau > \frac{\ell_o}{1+b}\right].$$

Note that this corresponds to $\varphi(\overline{\tau}, \ell_o)$. We know that $\varphi(\overline{\tau}, \ell_o) = \ell_o$ has a solution, namely $\widetilde{\ell}_r(\overline{\tau})$. For future reference, let $\widehat{\ell}_o = \widetilde{\ell}(\overline{\tau})$ denote this solution. We claim that $\widehat{\ell}_o > \ell_o^*$. This follows from the fact that

$$\ell_o^* = E\left[\tau|\tau > \frac{\ell_r^*+\ell_o^*}{2}\right]$$

and that

$$\frac{\ell^*+\ell_o^*}{2} < \frac{\frac{(1-b)\ell_o^*}{1+b}+\ell_o^*}{2} = \frac{\ell_o^*}{1+b}.$$

Next consider the equation

$$E\left[\tau|\frac{\ell_r}{1+b} < \tau < \frac{\ell_r}{1-b}\right].$$

We know that

$$\ell_r^* = E\left[\tau|\frac{\ell_r^*}{1+b} < \tau < \frac{\ell_r^*+\ell_o^*}{2}\right].$$

Furthermore, we know that

$$\frac{\ell_r^*+\ell_o^*}{2} > \frac{\ell_r^*+\frac{(1+b)\ell_r^*}{1-b}}{2} = \frac{\ell_r^*}{1-b}.$$

Thus

$$\ell_r^* > E\left[\tau | \frac{\ell_r^*}{1+b} < \tau < \frac{\ell_r^*}{1-b}\right].$$

On the other hand, as demonstrated in the proof of Proposition A3, we have that

$$\tau_m < E\left[\tau | \frac{\tau_m}{1+b} < \tau < \frac{\tau_m}{1-b}\right].$$

Thus by continuity there exists a $\widehat{\ell}_r \in (\tau_m, \ell^*)$ such that

$$\widehat{\ell}_r = E\left[\tau || \frac{\widehat{\ell}_r}{1+b} < \tau < \frac{\widehat{\ell}_r}{1-b}\right].$$

Note that

$$\widehat{\ell}_r < \ell^* < \frac{(1-b)\ell_o^*}{1+b} < \frac{(1-b)\widehat{\ell}_o}{1+b},$$

as required. Finally, we note that our solution $(\widehat{\ell}_r, \widehat{\ell}_o)$ is such that $(1+b)\underline{\tau} < \widehat{\ell}_r$. ■

8.7 Proof of Proposition A7

Under the condition on bias the citizen's welfare under a limit τ_m is given by

$$V(\tau_m) = \int_{\underline{\tau}}^{\tau_m} (\tau_m - \tau)h(\tau)d\tau - \int_{\tau_m}^{\overline{\tau}} (\tau - \tau_m)h(\tau)d\tau. \tag{A32}$$

Consider limits in the range $[\underline{\tau}, \tau_m)$. If $\ell + b \geq \overline{\tau}$ then the expression for the citizen's welfare in equation (A15) simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\overline{\tau}} (\tau - \ell)h(\tau)d\tau.$$

so $V(\tau_m)$ must exceed $V(\ell)$. If instead, $\ell + b < \overline{\tau}$, then the expresion for the citizen's welfare in equation (A15) is given by

$$V(\ell) = -\int_{\underline{\tau}}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\ell+b} (\tau - \ell)h(\tau)d\tau - \int_{\ell+b}^{\overline{\tau}} bh(\tau)d\tau.$$

Using this and equation (A32), we can write the benefit to the citizen of setting the limit at τ_m rather than ℓ as

$$\begin{aligned} V(\tau_m) - V(\ell) &= \int_{\underline{\tau}}^{\ell} (\ell - \tau_m) h(\tau) d\tau + \int_{\ell}^{\tau_m} (2\tau - \tau_m - \ell) h(\tau) d\tau \\ &\quad + \int_{\tau_m}^{\ell+b} (\tau_m - \ell) h(\tau) d\tau + \int_{\ell+b}^{\overline{\tau}} (\tau_m - \tau + b) h(\tau) d\tau \\ &= \int_{\underline{\tau}}^{\tau_m} (\ell - \tau_m) h(\tau) d\tau + 2 \int_{\ell}^{\tau_m} (\tau - \ell) h(\tau) d\tau \\ &\quad + \int_{\tau_m}^{\overline{\tau}} (\tau_m - \ell) h(\tau) d\tau + \int_{\ell+b}^{\overline{\tau}} (b - \tau + \ell) h(\tau) d\tau. \end{aligned}$$

Since h is symmetric,

$$V(\tau_m) - V(\ell) = 2 \int_{\ell}^{\tau_m} (\tau - \ell) h(\tau) d\tau + \int_{\ell+b}^{\overline{\tau}} (b - \tau + \ell) h(\tau) d\tau.$$

Since h is symmetric and non-decreasing on $[\underline{\tau}, \tau_m]$,

$$\begin{aligned} V(\tau_m) - V(\ell) &\geq 2 \int_{\ell}^{\tau_m} \left(\frac{\tau_m + \ell}{2} - \ell \right) h(\tau) d\tau + \int_{\ell+b}^{\overline{\tau}} \left(b - \frac{\overline{\tau} + \ell + b}{2} + \ell \right) h(\tau) d\tau \\ &\geq (\tau_m - \ell) \int_{\ell}^{\tau_m} h(\tau) d\tau - \frac{\overline{\tau} - (\ell + b)}{2} \int_{\ell+b}^{\overline{\tau}} h(\tau) d\tau. \end{aligned}$$

Since $\tau_m \geq \overline{\tau} - b$,

$$V(\tau_m) - V(\ell) \geq (\tau_m - \ell) \int_{\ell}^{\tau_m} h(\tau) d\tau - \frac{\tau_m - \ell}{2} \int_{\ell+b}^{\overline{\tau}} h(\tau) d\tau.$$

Since $b \geq \overline{\tau} - \tau_m$ we must have that $\tau_m - \ell \geq \overline{\tau} - (\ell + b)$. As a result,

$$\int_{\ell}^{\tau_m} h(\tau) d\tau \geq \int_{\ell+b}^{\overline{\tau}} h(\tau) d\tau,$$

and $V(\tau_m) - V(\ell) > 0$.

For limits in the range $(\tau_m, \overline{\tau}]$, if $\ell - b \geq \underline{\tau}$ then the expression for the citizen's welfare in equation (A15) simplifies to

$$V(\ell) = - \int_{\underline{\tau}}^{\ell} (\ell - \tau) h(\tau) d\tau - \int_{\ell}^{\overline{\tau}} (\tau - \ell) h(\tau) d\tau,$$

and $V(\tau_m)$ exceeds $V(\ell)$ from the argument in Section 7.1 of the On-line Appendix.

If instead $\ell - b > \underline{\tau}$ then the expression for the citizen's welfare in equation (A15) simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell-b} bh(\tau)d\tau - \int_{\ell-b}^{\ell} (\ell-\tau)h(\tau)d\tau - \int_{\ell}^{\overline{\tau}} (\tau-\ell)h(\tau)d\tau.$$

Using this and equation (A32), the benefit to the citizen of setting the limit at τ_m rather than ℓ is given by

$$V(\tau_m)-V(\ell) = \int_{\underline{\tau}}^{\ell-b} (\tau-\tau_m+b)h(\tau)d\tau + \int_{\ell-b}^{\tau_m} (\ell-\tau_m)h(\tau)d\tau + \int_{\tau_m}^{\ell} (\tau_m+\ell-2\tau)h(\tau)d\tau + \int_{\ell}^{\overline{\tau}} (\tau_m-\ell)h(\tau)d\tau,$$

which we can simplify to

$$V(\tau_m) - V(\ell) = \int_{\underline{\tau}}^{\tau_m} (\ell-\tau_m)h(\tau)d\tau + \int_{\underline{\tau}}^{\ell-b} (\tau+b-\ell)h(\tau)d\tau + \int_{\tau_m}^{\overline{\tau}} (\tau_m-\ell)h(\tau)d\tau + \int_{\tau_m}^{\ell} 2(\ell-\tau)h(\tau)d\tau$$

and

$$V(\tau_m) - V(\ell) = \int_{\underline{\tau}}^{\ell-b} (\tau+b-\ell)h(\tau)d\tau + 2\int_{\tau_m}^{\ell} (\ell-\tau)h(\tau)d\tau.$$

Since h is symmetric and non-decreasing on $[\underline{\tau}, \tau_m]$

$$V(\tau_m) - V(\ell) \geq \left(\frac{\ell-b+\underline{\tau}}{2} + b - \ell\right) \int_{\underline{\tau}}^{\ell-b} h(\tau)d\tau + 2(\ell - \frac{\ell+\tau_m}{2}) \int_{\tau_m}^{\ell} h(\tau)d\tau,$$

which we can simplify to

$$V(\tau_m) - V(\ell) \geq (\ell-\tau_m) \int_{\tau_m}^{\ell} h(\tau)d\tau - \frac{\ell-\underline{\tau}-b}{2} \int_{\underline{\tau}}^{\ell-b} h(\tau)d\tau.$$

We need to show that this expression exceeds zero. Note that since $b > \tau_m - \underline{\tau}$ it suffices to show that

$$(\ell-\tau_m) \int_{\tau_m}^{\ell} h(\tau)d\tau > \frac{\ell-\tau_m}{2} \int_{\underline{\tau}}^{\ell-b} h(\tau)d\tau.$$

Since b exceeds $\tau_m - \underline{\tau}$, it must be the case that $\ell - \tau_m$ exceeds $\ell - b - \underline{\tau}$ so this inequality holds. ■

8.8 Proof of Lemma A2

Proposition A7 implies that $b < \tau_m - \underline{\tau}$. First we show that it must be the case that $\ell - b \geq \underline{\tau}$ and $\ell + b \leq \underline{\tau}$.

If $\ell - b$ was less than $\underline{\tau}$ then the expression for the citizen's expected welfare in equation (A15) simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\ell+b} (\tau - \ell)h(\tau)d\tau - \int_{\ell+b}^{\overline{\tau}} bh(\tau)d\tau.$$

The welfare impact of a small change in the limit is given by

$$\begin{aligned} V'(\ell) &= -\int_{\underline{\tau}}^{\ell} h(\tau)d\tau + \int_{\ell}^{\ell+b} h(\tau)d\tau \\ &= H(\ell + b) - 2H(\ell). \end{aligned}$$

Note that since $\ell + b < \overline{\tau}$, $\ell < \tau_m$, and h is non-decreasing over the interval $[\underline{\tau}, \tau_m]$ and symmetric, we must have that

$$H(\ell + b) - H(\ell) \geq H(\underline{\tau} + b) - H(\underline{\tau}) = H(\underline{\tau} + b).$$

Since $\ell - b < \underline{\tau}$, ℓ must be less than $\underline{\tau} + b$. Hence,

$$H(\ell + b) - H(\ell) > H(\ell),$$

and a small increase in the limit would improve the voter's welfare. If $\ell + b$ exceeded $\overline{\tau}$ then the expression for the citizen's expected welfare in equation (A15) simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell-b} bh(\tau)d\tau - \int_{\ell-b}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\overline{\tau}} (\tau - \ell)h(\tau)d\tau.$$

The welfare impact of a small change in the limit is given by

$$\begin{aligned} V'(\ell) &= -\int_{\ell-b}^{\ell} h(\tau)d\tau + \int_{\ell}^{\overline{\tau}} h(\tau)d\tau \\ &= 1 - 2H(\ell) + H(\ell - b). \end{aligned}$$

Note that since $\ell - b > \underline{\tau}$, $\ell > \tau_m$, and h is non-increasing over the interval $[\tau_m, \underline{\tau}]$ and symmetric, we must have that

$$H(\ell) - H(\ell - b) \geq H(\overline{\tau}) - H(\overline{\tau} - b) = 1 - H(\overline{\tau} - b).$$

Since $\ell + b > \overline{\tau}$, ℓ must be greater than $\overline{\tau} - b$. Hence,

$$H(\ell) - H(\ell - b) > 1 - H(\ell),$$

and a small decrease in the limit would improve the voter's welfare.

Thus, for any candidate optimal limit we have that $\ell - b \geq \underline{\tau}$ and $\ell + b \leq \overline{\tau}$. The expression for the citizen's expected welfare in equation (A15) then simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell-b} b\tau h(\tau)d\tau - \int_{\ell-b}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\ell+b} (\tau - \ell)h(\tau)d\tau - \int_{\ell+b}^{\overline{\tau}} b\tau h(\tau)d\tau.$$

Hence, the effect of a small change in the limit on the citizen's expected welfare is

$$V'(\ell) = 2H(\ell) - H(\ell - b) - H(\ell + b).$$

Since h is symmetric and non-decreasing over $[\underline{\tau}, \tau_m]$, if $\ell < \tau_m$ and h is not constant over the interval $[\ell - b, \ell + b]$, then $H(\ell + b) - H(\ell)$ must exceed $H(\ell) - H(\ell - b)$ and a small increase in the limit would improve the voter's welfare.

Since h is symmetric and non-decreasing over $[\underline{\tau}, \tau_m]$, if $\ell > \tau_m$ and h is not constant over the interval $[\ell - b, \ell + b]$, then $H(\ell) - H(\ell - b)$ must exceed $H(\ell + b) - H(\ell)$ and a small decrease in the limit would improve the voter's welfare. ■

8.9 Proof of Proposition A8

Suppose that $\ell \neq \tau_m$ is in the set of optimal limits and τ_m is not. Lemma A2 implies that $\ell - b$ must exceed $\underline{\tau}$ and $\ell + b$ must be less than $\underline{\tau}$. As a result, we have that the expression for the citizen's expected welfare in equation (A15), under either limit, simplifies to

$$V(\ell) = -\int_{\underline{\tau}}^{\ell-b} b\tau h(\tau)d\tau - \int_{\ell-b}^{\ell} (\ell - \tau)h(\tau)d\tau - \int_{\ell}^{\ell+b} (\tau - \ell)h(\tau)d\tau - \int_{\ell+b}^{\overline{\tau}} b\tau h(\tau)d\tau.$$

Thus, we can write the difference in the citizen's expected welfare under the two limits as

$$\begin{aligned}V(\tau_m)-V(\ell)=&\int_{\ell-b}^{\ell}(\ell-\tau)h(\tau)d\tau+\int_{\ell}^{\ell+b}(\tau-\ell)h(\tau)d\tau+\int_{\ell+b}^{\tau_m+b}bh(\tau)d\tau\\&-\int_{\ell-b}^{\tau_m-b}bh(\tau)d\tau-\int_{\tau_m-b}^{\tau_m}(\tau_m-\tau)h(\tau)d\tau-\int_{\tau_m}^{\tau_m+b}(\tau-\tau_m)h(\tau)d\tau.\end{aligned}$$

Lemma A2 implies that h is constant over the interval $[\ell-b,\ell+b]$ so,

$$\begin{aligned}V(\tau_m)-V(\ell)=&\int_{\ell-b}^{\ell}\left(\ell-\frac{\ell+\ell-b}{2}\right)h(\tau)d\tau+\int_{\ell}^{\ell+b}\left(\frac{\ell+b+\ell}{2}-\ell\right)h(\tau)d\tau+\int_{\ell+b}^{\tau_m+b}bh(\tau)d\tau\\&-\int_{\ell-b}^{\tau_m-b}bh(\tau)d\tau-\int_{\tau_m-b}^{\tau_m}(\tau_m-\tau)h(\tau)d\tau-\int_{\tau_m}^{\tau_m+b}(\tau-\tau_m)h(\tau)d\tau.\end{aligned}$$

Which using the same property can be simplified to

$$\begin{aligned}V(\tau_m)-V(\ell)=&b\int_{\ell}^{\ell+b}h(\tau)d\tau+\int_{\ell+b}^{\tau_m+b}bh(\tau)d\tau\\&-\int_{\ell-b}^{\tau_m-b}bh(\tau)d\tau-\int_{\tau_m-b}^{\tau_m}(\tau_m-\tau)h(\tau)d\tau-\int_{\tau_m}^{\tau_m+b}(\tau-\tau_m)h(\tau)d\tau.\end{aligned}$$

Since h is non-decreasing over the interval $[\underline{\tau},\tau_m]$ and is symmetric

$$\begin{aligned}V(\tau_m)-V(\ell)\geq&b\int_{\ell}^{\ell+b}h(\tau)d\tau+\int_{\ell+b}^{\tau_m+b}bh(\tau)d\tau\\&-\int_{\ell-b}^{\tau_m-b}bh(\tau)d\tau-\int_{\tau_m-b}^{\tau_m}\left(\tau_m-\frac{2\tau_m-b}{2}\right)h(\tau)d\tau-\int_{\tau_m}^{\tau_m+b}\left(\frac{2\tau_m+b}{2}-\tau_m\right)h(\tau)d\tau.\end{aligned}$$

Using the same properties and simplifying yields

$$V(\tau_m)-V(\ell)\geq b\left[\int_{\ell}^{\ell+b}h(\tau)d\tau+\int_{\ell+b}^{\tau_m+b}h(\tau)d\tau-\int_{\ell-b}^{\tau_m-b}h(\tau)d\tau-\int_{\tau_m-b}^{\tau_m}h(\tau)d\tau\right].$$

First suppose that $b<|\tau_m-\ell|/2$. Hence either $\ell<\tau_m$ and

$$\underline{\tau}<\ell-b<\ell<\ell+b<\tau_m-b<\tau_m<\tau_m+b<\overline{\tau}$$

or $\ell>\tau_m$ and

$$\underline{\tau}<\tau_m-b<\tau_m<\tau_m+b<\ell-b<\ell<\ell+b.$$

In either case we have that

$$V(\tau_m) - V(\ell) \geq b\left[\int_{\tau_m}^{\tau_m+b} h(\tau)d\tau - \int_{\ell-b}^{\ell} h(\tau)d\tau\right].$$

If ℓ is less than τ_m then since h is non-decreasing on the interval $[\underline{\tau}, \tau_m]$ and symmetric this is at least zero. If ℓ exceeds τ_m then since h is non-increasing on the interval $[\tau_m, \overline{\tau}]$ this is at least zero.

Next suppose that $\frac{|\tau_m-\ell|}{2} < b < |\tau_m - \ell|$. Hence if $\ell < \tau_m$ then

$$\underline{\tau} < \ell - b < \ell < \tau_m - b < \ell + b < \tau_m < \tau_m + b$$

and

$$V(\tau_m) - V(\ell) \geq b\left[\int_{\tau_m-b}^{\tau_m} h(\tau)d\tau - \int_{\ell-b}^{\ell} h(\tau)d\tau\right].$$

Since h is non-decreasing over the interval $[\ell - b, \tau_m]$, this is greater than or equal to zero.

If instead $\ell > \tau_m$ then

$$\underline{\tau} < \tau_m - b < \tau_m < \ell - b < \tau_m + b < \ell < \ell + b$$

and

$$V(\tau_m) - V(\ell) \geq \left[\int_{\tau_m}^{\ell-b} h(\tau)d\tau - \int_{\tau_m+b}^{\ell} h(\tau)d\tau\right].$$

Since h is non-decreasing over the interval $[\ell - b, \tau_m]$, this is greater than or equal to zero.

Lastly suppose that $b > |\tau_m - \ell|$. Hence either $\ell < \tau_m$ and

$$\underline{\tau} < \ell - b < \tau_m - b < \ell < \tau_m < \ell + b < \tau_m + b < \overline{\tau}$$

or $\ell > \tau_m$ and

$$\underline{\tau} < \tau_m - b < \ell - b < \tau_m < \ell < \tau_m + b < \ell + b < \overline{\tau}.$$

In either case we have that

$$V(\tau_m) - V(\ell) = \int_{\ell+b}^{\tau_m+b} bh(\tau)d\tau + \int_{\ell-b}^{\ell} (\ell-\tau)h(\tau)d\tau + \int_{\ell}^{\ell+b} (\tau-\ell)h(\tau)d\tau$$
$$+ \int_{\tau_m-b}^{\tau_m} (\tau-\tau_m)h(\tau)d\tau + \int_{\tau_m}^{\tau_m+b} (\tau_m-\tau)h(\tau)d\tau - \int_{\ell-b}^{\tau_m-b} bh(\tau)d\tau.$$

Since h is symmetric and constant over the interval $[\ell-b, \ell+b]$ it must also be constant over the interval $[\ell-b, \tau_m+b]$. As a result this difference is equal to zero. ■

8.10 Proof of Proposition A10

There are four possibilities for the optimal system of limits (ℓ_r, ℓ_o): i) $\ell_r \leq \underline{\tau}+b$ and $\ell_r + b \geq \ell_o - b$; ii) $\ell_r > \underline{\tau}+b$ and $\ell_r + b \geq \ell_o - b$; iii) $\ell_r > \underline{\tau}+b$ and $\ell_r + b < \ell_o - b$; iv) $\ell_r \leq \underline{\tau}+b$ and $\ell_r + b < \ell_o - b$.

We begin by showing that if the politician's bias is below $(\overline{\tau}-\underline{\tau})/8$, cases i) and iv) are not possible. We first rule out case i). By the argument in the Proof of Proposition 7, any solution must have $\ell_r \geq \frac{3\underline{\tau}+\tau_m}{2}$. Thus, we cannot have this solution if $\underline{\tau}+b < \frac{3\underline{\tau}+\tau_m}{4}$, which is equivalent to $b < \frac{\tau_m-\underline{\tau}}{4} = \frac{\overline{\tau}-\underline{\tau}}{8}$.

To rule out case iv) note that if $\ell_r \in (\underline{\tau}, \tau_m)$ then since h is non-decreasing over the interval $(\underline{\tau}, \ell_r)$ and $\ell_r - \underline{\tau} < b$ it must be the case that

$$H(\ell_r) < H(\ell_r+b) - H(\ell_r).$$

Thus, a small increase in the limit would improve the voter's welfare. This implies that $\ell_r \in [\tau_m, \overline{\tau}]$. Since $\ell_r - b \leq \underline{\tau}$ it must also be the case that $\ell_r + b \geq \overline{\tau}$. Hence there is no $\ell_o \in [\underline{\tau}, \overline{\tau}]$ for which $\ell_r + b < \ell_o - b$.

We next show that there exists a solution to system (A25). For $x \in [\underline{\tau}, \overline{\tau}]$, let $\ell_o(x)$ be defined by

$$1 - H(\ell_o) = H(\ell_o) - H(x).$$

As argued in the proof of assertions made after Proposition 6, $\ell_o(x)$ is well defined and increases smoothly from τ_m to $\overline{\tau}$. Now define function

$$\varphi(\ell_r) = 2H(\ell_r) - H(\ell_r - b).$$

Note that this function is continuous and increasing on $[\underline{\tau}, \tau_m]$ and is such that $\varphi(\underline{\tau}) = 0$ and $\varphi(\overline{\tau}) > 1$. Furthermore, because

$$\varphi'(\ell_r) = 2h(\ell_r) - h(\ell_r - b).$$

and h is non-increasing on $[\tau_m, \overline{\tau}]$, $\varphi(\ell_r)$ is not necessarily increasing on $(\tau_m, \overline{\tau}]$. Consider then for all $x \in [\underline{\tau}, \overline{\tau}]$ the equation

$$\varphi(\ell_r) = H(x).$$

Since $\varphi(\underline{\tau} = 0)$, $\varphi(\overline{\tau}) > 1$, and $\varphi(\ell_r)$ is continuous, this equation has a solution for all x, and it may have multiple solutions because $\varphi(\ell_r)$ is not necessarily increasing on $(\tau_m, \overline{\tau}]$. Let $\widetilde{\ell}_r(x)$ be defined as the smallest such solution. Note that $\widetilde{\ell}_r(x)$ will be increasing and continuous almost everywhere. However, it may have a discrete number of upward jumps. Finally, because $\varphi(\ell_r)$ is continuous and increasing on $[\underline{\tau}, \tau_m]$, $\widetilde{\ell}_r(x)$ will be continuous on $[\underline{\tau}, x^*]$ where x^* is such that $\varphi(\tau_m) = H(x^*)$.

Define the function on the interval $[\underline{\tau}, \overline{\tau}]$ as follows

$$f(x) = \frac{\widetilde{\ell}(x) + \ell_o(x)}{2}.$$

Solutions to system (A25) correspond to values of x such that $f(x) = x$. Note that $f(x)$ is increasing, continuous on $[\underline{\tau}, x^*)$, and continuous almost everywhere. Moreover, $f(\underline{\tau}) > \underline{\tau}$ and $f(\overline{\tau}) < \overline{\tau}$. Thus by the argument in the Proof of Proposition 7, there must exist some x such that $f(x) = x$.

It follows that system (A25) has a solution. Let (ℓ^*, ℓ_o^*) be such a solution. We can verify that $\ell_r^* > \underline{\tau} + b$ using the argument in the Proof of Proposition 7 which implies that $\widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) > \frac{3\underline{\tau}+\tau_m}{4}$. The fact that $b < \frac{\overline{\tau}-\underline{\tau}}{8}$ implies that $\underline{\tau} + b < \frac{3\underline{\tau}+\tau_m}{4} < \widetilde{\ell}_r(\frac{\underline{\tau}+\tau_m}{2}) \leq \ell_r^*$.

Next we show that if at the solution (ℓ_r^*, ℓ_o^*), we have that $\ell_r^* + b < \ell_o^* - b$ it must be the case that system (A26) has a solution in which $\ell_r + b < \ell_o - b$. Consider first the equation

$$H(\ell_o) - H(\ell_o - b) = 1 - H(\ell_o).$$

Note that this corresponds to $\varphi(\ell_o) = H(\overline{\tau})$. We know that this equation has a solution: namely, $\widetilde{\ell}_r(\overline{\tau})$. For future reference, let $\widehat{\ell}_o = \widetilde{\ell}_r(\overline{\tau})$ denote this solution. We claim that $\widehat{\ell}_o > \ell_o^*$. This follows from the facts that

$$1 - H(\ell_o^*) = H(\ell_o^*) - H(\frac{\ell_r^* + \ell_o^*}{2})$$

and

$$\frac{\ell_r^* + \ell_o^*}{2} < \frac{\ell_o^* + b + \ell_o^*}{2} = \ell_o - b,$$

so that

$$H(\frac{\ell^* + \ell_0^*}{2}) < H(\frac{\ell_o^*}{1+b}).$$

Next consider the equation

$$H(\ell) - H(\ell - b) = H(\ell + b) - H(\ell).$$

We know that

$$H(\ell^*) - H(\ell_r^* - b) = H(\frac{\ell_r^* + \ell_o^*}{2}) - H(\ell^*)$$

Furthermore, we know that

$$\frac{\ell_r^* + \ell_o^*}{2} > \frac{\ell_r^* + \ell_r^* + 2b}{2} = \ell_r^* + b.$$

Thus,

$$H(\ell^*) - H(\ell^* - b) > H(\ell^* + b) - H(\ell^*).$$

On the other hand we know that

$$H(\tau_m) - H(\tau_m - b) < H(\tau_m + b) - H(\tau_m).$$

Thus, by continuity, there exists a $\widehat{\ell}_r \in (\tau_m, \ell_r^*)$ such that

$$H(\widehat{\ell}_r) - H(\widehat{\ell}_r - b) = H(\widehat{\ell}_r + b) - H(\widehat{\ell}_r).$$

Note that

$$\widehat{\ell}_r < \ell_r^* < \ell_o^* - 2b < \widehat{\ell}_o^* - 2b,$$

as required.

Finally, we note that our solution $(\widehat{\ell}_r, \widehat{\ell}_o)$ is such that $\widehat{\ell}_r - b > \underline{\tau}$. ■

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